HISTORY OF ALGEBRAIC GEOMETRY

Time and location: Fridays, 12-2pm, Math 528.
Section instructor: Caleb Ji (cj2670@columbia.edu)
Course website: www.math.columbia.edu/~calebji/teaching/historyag.html

1. Description of course

One way to categorize the fundamental notions of mathematics is through the notions of equations (algebra), space (geometry), measure (analysis), and number (number theory). Throughout the development of mathematics, especially in the last 200 years, these notions gradually liberated themselves from the strict confines of their original conception. In modern algebra, we now study abstract structures such as groups, rings, and fields that appear in all sorts of mathematical situations. In geometry, space is no longer restricted to the Euclidean space our minds naturally conceive of, not even if we give it an arbitrary number of dimensions. Instead, it is encapsulated by the notion of a manifold, a topological space which 'looks like' a manifold at each point.

The historical roots of algebraic geometry lie in studying the shape of the solutions to polynomial equations. Polynomials are the most basic kind of algebraic expressions – constructed simply from addition and multiplication. Restricting to these gives a very interesting class of manifolds. But as both algebra and geometry evolved immensely through the years, algebraic geometry did as well, and came to incorporate fields such as complex analysis and number theory in essential ways.

In this course we will study algebraic geometry from a historical perspective, with a special emphasis on the period 1850 - 1975. We will begin with Riemann, whose groundbreaking study of Riemann surfaces which set the stage for a century of further developments. We will end with Grothendieck and Deligne, and their incredible proof of the Weil conjectures.

While it is not possible to give proper proofs in this course of even a fraction of the material we will cover, in the spirit of a history class we will still attempt to convey a portion of the progression of ideas in these topics. Thus, while we will generally proceed chronologically, when considering a given topic we will also look ahead for new language, simplifications, and progress that will help us to better understand the underlying ideas.

2. Grading and expectations

This class will meet once a week on Fridays. You are expected to attend and participate in every meeting. Unexcused absences may result in a deduction from your grade. Aside from participation, your grade will either be based on your two presentations, or your presentation and your paper.

Presentation. You will choose topics from the topics list below. If you want to present on something else, please get my approval first. Your presentation should last betwen 45 minutes and an hour. There will be two presenters each week, and you are encouraged to prepare with the person you are presenting with and find connections between your topics.

You are highly recommended to use the chalkboard, rather than slides. Your talk should primarily be a math talk, so you should be writing formulas, theorems, and maybe a few proof sketches.

Furthermore, every topic will involve some basic mathematical material that is part of the advanced undergraduate curriculum. You are encouraged to explain these parts in detail. You are also encouraged to sprinkle in relevant historical tidbits if you wish. Finally, you are also free to prepare problems for the class to discuss after you have finished your talk.

After you choose your topic, you are encouraged to meet with me to discuss how to get started. I will be able to provide you with references and give you general tips on presenting.

Paper. You will turn in an expository paper at any point before the end of the semester. You may choose to delve deeper into the topic you presented on or choose a different topic. The paper needs to be single-spaced, in LaTeX, and at least 5 pages long not counting references. You are again encouraged to weave in historical discussions in your paper as you see fit.

Grading. Due to the difficulty of the topics covered, this class is mostly graded off of effort. If you attend and participate in class and put in a solid effort into your presentations/paper, it is likely that you will receive an A. However, you will not be able to cram these in a few days. If you do, it will show in your work and your grade may suffer.

Most students will present twice. If you present twice you are still encouraged but not required to write an expository paper. If you do so, you will have a higher chance of receiving an A.

3. Textbook

This course is based on the book *History of Algebraic Geometry* by Jean Dieudonné. You are encouraged to use it as a reference for planning your presentation and paper. Ask me for other references specific to your presentation/paper.

4. Prerequisites

It is recommended, but not required, that you have taken at least one 400 level math class at Columbia.

5. Topics

We will not cover all of the following topics, but most of our topics will be drawn from this list.

(1) History up through 1850: singularities, degree, Bezout's theorem, projective geometry, enumerative geometry
(2) Riemann: Abel's theorem, complex analysis, abelian integrals, Riemann surfaces, birational geometry, Riemann-Roch, moduli of curves
(3) Kronecker, Dedekind, and Weber: commutative algebra, algebraic number theory, valuations, divisors, Riemann-Roch
(4) Picard, Poincaré, Lefschetz: algebraic topology, Picard-Fuchs equation, Picard groups, abelian varieties
(5) The Italian school: linear systems, genus and other cohomological invariants, Enriques-Kodaira classification of surfaces
(6) Kähler, Hodge: differential forms, Kähler manifolds, Hodge theory
(7) Hilbert: basis theorem, Nullstellensatz, syzygy theorem, Hilbert functions, invariant theory
(8) Zariski, Weil: local rings, Zariski topology, resolution of singularities, zeta functions, Weil conjectures
(9) Serre, Grothendieck: coherent sheaves, sheaf cohomology, schemes, Grothendieck-Riemann-Roch
(10) Grothendieck, Deligne: étale cohomology, proof of the Weil conjectures, motives