Calculus I Practice Midterm Exam

Instructions

- Write **your name and UNI** clearly in the section below.

- You are **NOT** allowed to use class notes, books and homework solutions in the examination.

- Except for True/False questions, show all computations and work in your answer.

- **Don't cheat**! If it looks like you are cheating, then you are cheating.

Question	Points	Score
1	10	
2	10	
3	4	
4	6	
5	10	
6	5	
7	5	
Total:	50	

Name:_____

UNI:_____

1. (10 points) **True/False** 2 points each

   (a) $\boxed{\text{T}}$ $\boxed{\text{F}}$ $f(x) = \sin(x^2)$ is an even function.

   (b) $\boxed{\text{T}}$ $\boxed{\text{F}}$ The graph of $f(2x)$ is obtained from stretching the graph of $f(x)$ horizontally by a factor of 2.

   (c) $\boxed{\text{T}}$ $\boxed{\text{F}}$ We have that

   $$\lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = \lim_{x \to 0} x \cdot \lim_{x \to 0} \sin\left(\frac{1}{x}\right)$$

   (d) $\boxed{\text{T}}$ $\boxed{\text{F}}$ The function $f(x) = x^6 + x - 1$ has a solution in $(0, 1)$.

   (e) $\boxed{\text{T}}$ $\boxed{\text{F}}$ The derivative of 1 is 1.

   (You may use this area as scratchwork.)

2. Compute the following limits, if they exist. If the limit does not exist, explain why.

(a) (3 points) $\displaystyle\lim_{x\to 3} \frac{x-2}{x^2 - 5x + 6}$

(b) (3 points) $\displaystyle\lim_{x\to 0} x^4 \sin\left(\frac{1}{x}\right)$

(c) (4 points) $\displaystyle\lim_{x\to 0} \cos\left(\frac{\sqrt{2+x} - \sqrt{2-x}}{x}\right)$

3. Please give formal definitions below.

   (a) (2 points) What does it mean for a function $f(x)$ to be continuous at a point a?

   (b) (2 points) What does it mean for a function $f(x)$ to be differentiable at a point a?

4. Consider the following function.

$$f(x) = \begin{cases} 2 & \text{if } x \leq -1 \\ 10 - x^2 & \text{if } -1 < x < 3 \\ \dfrac{1}{4 - x} & \text{if } x \geq 3 \end{cases}$$

   (a) (3 points) For what values of x is f not continuous at x?

   (b) (3 points) For what values of x is f not differentiable at x?

5. Compute the value of the derivative of $f(x)$ at the point a. If $f(x)$ is not differentiable at a, explain why.

   (a) (3 points) $f(x) = x^3 + \sqrt{x}$, $a = 4$

   (b) (3 points) $f(x) = \dfrac{7}{x^6}$, $a = 1$

   (c) (4 points) $f(x) = 2|x - 3|$, $a = 3$

6. (5 points) Find an equation of the tangent line to the curve $y = 3x^3 + 2x^2 + 1$ at the point $(-1, 0)$.

7. (5 points) Find all vertical and horizontal asymptotes of the graph of $f(x) = \dfrac{\sqrt{9x^2 + 3}}{4x - 1}$.