Elliptic Curves.

lattice $L \subset \mathbb{C}$, $(\mathbb{C}/L, \omega = dz)$.

Weierstrass $z \in \mathbb{C}/L \longrightarrow (x = \wp(z, L), \; y = \wp'(z, L))$

$$y^2 = 4x^3 - g_2 x - g_3$$

$$\omega \longmapsto \frac{dx}{y}$$

Conversely, $E/\mathbb{C}$ elliptic curve, ω nonzero holo. differential, the lattice

$$L(E, \omega) = \left\{ \int_\gamma \omega \;\middle|\; \gamma \in H_1(E, \mathbb{Z}) \right\} \subset \mathbb{C}$$

$\forall \lambda \in \mathbb{C}^\times$, $L(E, \lambda\omega) = \lambda L(E, \omega)$

Modular form f on $\mathcal{H}$ of level 1, wt k defines a function F of lattices

$F(L)$ s.t. $f(\tau) = F(\mathbb{Z}\tau + \mathbb{Z})$, $F(\lambda L) = \lambda^{-k} F(L)$, $\lambda \in \mathbb{C}^\times$.

$\left(F(\mathbb{Z}w_1 + \mathbb{Z}w_2) = w_2^{-k} f\left(\frac{w_1}{w_2}\right), \quad \mathrm{Im}\, \frac{w_1}{w_2} > 0 \right)$.

Then f, F gives a function on pairs (E, ω), $E/\mathbb{C}$ elliptic curve, ω nowhere vanishing differential, $F(E, \omega) = F(L(E, \omega))$ s.t. $F(E, \lambda\omega) = \lambda^{-k} F(E, \omega)$.

Holomorphy at ∞.

$q = e^{2\pi i \tau}$, $t = e^{2\pi i z}$, $(\mathbb{C}/2\pi i\mathbb{Z} + 2\pi i \mathbb{Z}\tau, \; 2\pi i\, dz) \simeq (\mathbb{C}^*/q^{\mathbb{Z}}, \frac{dt}{t})$

$$\wr$$

$$C: Y^2 = 4X^3 - \frac{E_4(\tau)}{12} X + \frac{E_6(\tau)}{216}, \quad \omega = \frac{dX}{Y}$$

$$X = \wp(2\pi i z, 2\pi i\mathbb{Z} + 2\pi i \mathbb{Z}\tau), \quad Y = \wp'(2\pi i z, 2\pi i \mathbb{Z} + 2\pi i\mathbb{Z}\tau)$$

Modular form f be mero. / holo. at ∞ is to ask $F(C, \omega) \in \mathbb{C}((q))$
or $\mathbb{C}[[q]]$.

Let $X = x + \frac{1}{12}$, $Y = x + 2y$ $\Rightarrow$ $\mathrm{Tate}(q)$: $y^2 + xy = x^3 + B(q)x + C(q)$

$$B(q) = -5 \frac{E_4 - 1}{240} = -5 \sum_{n \geq 1} \sigma_3(n) q^n$$

$\in \mathbb{Z}[[q]]$

$$C(q) = \frac{1}{12}\left(-5 \frac{E_4 - 1}{240} - 7 \frac{E_6 - 1}{-504}\right) = \sum_{n \geq 1} \frac{-5\sigma_3(n) - 7\sigma_5(n)}{12} q^n$$

$\omega_{can} = \frac{dy}{2y + x}$. $(\mathrm{Tate}(q), \omega_{can})$ is an elliptic curve over $\mathbb{Z}((q))$, and will be used to provide q-expansion of modular forms.

Explicitly $t = e^{2\pi i z}$, $x(t) = \sum_{k \in \mathbb{Z}} \frac{q^k t}{(1 - q^k t)^2} - 2 \sum_{k=1}^{\infty} \frac{q^k}{1 - q^k}$

$$y(t) = \sum_{k \in \mathbb{Z}} \frac{(q^k t)^2}{(1 - q^k t)^3} + \sum_{k=1}^{\infty} \frac{q^k}{1 - q^k}$$

group law : $(x(t), y(t)) \oplus (x(s), y(s)) = (x(ts), y(ts))$.

Tate curve $(\mathrm{Tate}(q^n), \omega_{can})$, $y^2 + xy = x^3 + B(q^n)x + C(q^n)$, $\omega = \frac{dx}{2y + x}$.

ζ_n be primitive n-th roots of unity, points of $\mathbb{C}^* / q^{n\mathbb{Z}}$ of order n are

$\zeta_n^i q^j$, $0 \leq i, j \leq n-1$

each of the n^2-1 points $\zeta_n^i q^j$. $(i,j) \neq (0,0)$ has x, y coordinates in $\mathbb{Z}[[q]] \underset{\mathbb{Z}}{\otimes} \mathbb{Z}[\zeta_n, \frac{1}{n}] \subsetneq \mathbb{Z}[\zeta_n, \frac{1}{n}][[q]]$.

Moduli Schemes & q-expansions.

An elliptic curve $p: E \to S$ is a proper smooth map, whose geometric fibres are connected curves of genus 1, together with a section $e: S \to E$.

$\omega_{E/S} = p_* \Omega^1_{E/S}$ invertible sheaf on S, canonically dual to $R^1 p_* \mathcal{O}_E$.

A modular form of wt $k \in \mathbb{Z}$ and level one is f assigns to every E/S elliptic curve, a section $f(E/S) \in \Gamma(S, \omega_{E/S}^{\otimes k})$ depending only on S-isom. class of E/S and commutes with base change $S' \to S$.

Equivalently f assigns every pair $(E/R, \omega)$. E/R elliptic curve, ω base of $\omega_{E/R}$ (nowhere vanishing section of $\Omega^1_{E/S}$ on E), $f(E/R, \omega) \in R$, depending only on R-isom. class of E/R, commutes with base change and

$\forall \lambda \in R^\times$, $f(E/R, \lambda\omega) = \lambda^{-k} f(E/R, \omega)$.

$(f(E/R) = f(E/R, \omega) \cdot \omega^{\otimes k})$

Fix ground ring R_0, and consider in R_0-schemes. All modular forms of wt k, level one over R_0 forms R_0-module $M(R_0, 1, k)$.

$\forall f \in M(R_0, 1, k)$, $f((\mathrm{Tate}(q), \omega_{can})_{R_0}) \in \mathbb{Z}((q)) \otimes_{\mathbb{Z}} R_0$ called q-expansion of f. f called holo. at ∞ if $\cdots \in \mathbb{Z}[[q]] \otimes_{\mathbb{Z}} R_0$, all such forms give a submodule $S(R_0, 1, k)$.

Level n modular forms.

$\forall n \geq 1$, $E[n] = \ker(E \xrightarrow{[n]} E)$ is finite flat commutative group scheme of rank n^2 over S, etale over S iff n invertible on S, i.e. $S / \mathbb{Z}[\frac{1}{n}]$. A level n structure on E/S is an isom. $\alpha_n : E[n] \xrightarrow{\sim} (\mathbb{Z}/n\mathbb{Z})_S^2$, possible only when n invertible on S.

A modular form of wt k, level n. f assigns to each pair $(E/S, \alpha_n)$ E/S elliptic curve, α_n a level n structure on E/S, $f(E/S, \alpha_n) \in \Gamma(S, \omega_{E/S}^{\otimes k})$ depending $\cdots$ commuting $\cdots$ Equivalently $\cdots$ $\Rightarrow$ R_0-module $M(R_0, n, k)$.

If $\frac{1}{n}, \zeta_n \in R_0$, $\forall f \in M(R_0, n, k)$, $f((\mathrm{Tate}(q^n), \omega_{can}, \alpha_n)_{R_0}) \in \mathbb{Z}((q)) \otimes_{\mathbb{Z}} R_0$ called q-expansions of f (runs over all α_n)

$f \in M(R_0, n, k)$ is called holo. at ∞ if its inverse image on $R_0[\frac{1}{n}, \zeta_n]$ has all q-expansions in $\mathbb{Z}[[q]] \otimes_{\mathbb{Z}} R_0[\frac{1}{n}, \zeta_n]$. All denoted $S(R_0, n, k)$.

The modular schemes M_n, $\overline{M}_n$.

For each $n \geq 3$, the functor " Isom. class of elliptic curves with level n " is representable by M_n.

M_n affine smooth curve $/ \mathbb{Z}[\frac{1}{n}]$, finite flat of deg $\# GL_2(\mathbb{Z}/n\mathbb{Z})/\pm 1$ over affine j-line $\mathbb{Z}[\frac{1}{n}, j]$, etale over the open where j, $j-1728$ invertible.

$\overline{M}_n$ normalization of projective j-line $\mathbb{P}^1_{\mathbb{Z}[\frac{1}{n}]}$ in M_n, proper smooth curve $/ \mathbb{Z}[\frac{1}{n}]$,

$\Gamma(\overline{M}_n, \mathcal{O}_{\overline{M}_n}) = \mathbb{Z}[\frac{1}{n}, \zeta_n]$.

$\overline{M}_n \otimes \cdots$ proper

$M_n \otimes_{\mathbb{Z}[\frac{1}{n}]} \mathbb{Z}[\frac{1}{n}, \zeta_n]$ is disjoint union of $\varphi(n)$ affine smooth geometrically connected curves $/ \mathbb{Z}[\frac{1}{n}, \zeta_n]$.

$\overline{M}_n - M_n / \mathbb{Z}[\frac{1}{n}]$ is finite etale, over $\mathbb{Z}[\frac{1}{n}, \zeta_n]$ is disjoint union of sections called cusps of $\overline{M}_n$, which in natural correspondence to all Isom. class of level n structures on $\mathrm{Tate}(q^n)$ over $\mathbb{Z}((q)) \otimes_{\mathbb{Z}} \mathbb{Z}[\frac{1}{n}, \zeta_n]$.

The completion of $\overline{M}_n \otimes \mathbb{Z}[\frac{1}{n}, \zeta_n]$ along any of the cusps is $\simeq \mathbb{Z}[\frac{1}{n}, \zeta_n][[q]]$.

$\mathbb{P}^1_{\mathbb{Z}[\frac{1}{n}, \zeta_n]}$ ∞ $\simeq \mathbb{Z}[\frac{1}{n}, \zeta_n][[q]]$

and $\overline{M}_n \longrightarrow \mathbb{P}^1$ gives $\mathbb{Z}[\frac{1}{n}, \zeta_n][[q]] \longrightarrow \mathbb{Z}[\frac{1}{n}, \zeta_n][[q]]$

$$q \longmapsto q^n$$

For each cusp, the inverse image of the universal elliptic curve $(E/M_n, \alpha_n)$ over $\mathbb{Z}[\frac{1}{n}, \zeta_n]((q))$ is isom. to the inverse image of $\mathrm{Tate}(q^n)$ with the level n structure corresponding to that cusp over $\mathbb{Z}[\frac{1}{n}, \zeta_n]((q))$.

There is a unique invertible sheaf ω on $\overline{M_n}$, $\omega|_{M_n} = \omega_{E/M_n}$, E/M_n universal elliptic curve and the sections of ω over the completion $\mathbb{Z}[\frac{1}{n}, \zeta_n][[q]]$ at each cusp are $\mathbb{Z}[\frac{1}{n}, \zeta_n][[q]] \cdot \omega_{can}$. The Kodaira-Spencer style isom.

$$\omega_{E/M_n}^{\otimes 2} \xrightarrow{\sim} \Omega^1_{M_n/\mathbb{Z}[\frac{1}{n}]} \quad \text{extends to an isom,}$$

$$\omega^{\otimes 2} \xrightarrow{\sim} \Omega^1_{\overline{M_n}/\mathbb{Z}[\frac{1}{n}]}(\log(\overline{M_n} - M_n))$$

A modular form of wt n, level n, holo. at ∞ defined over $R_0 \ni \frac{1}{n}$ is just a section of $\omega^{\otimes k}$ on $\overline{M_n} \underset{\mathbb{Z}[\frac{1}{n}]}{\otimes} R_0$ or $\omega^{\otimes k} \underset{\mathbb{Z}[\frac{1}{n}]}{\otimes} R_0$ on $\overline{M_n}$.

i.e. $S(R_0, n, k) = H^0(\overline{M_n}, \omega^{\otimes k} \underset{\mathbb{Z}[\frac{1}{n}]}{\otimes} R_0)$

q-expansion principle.

K $\mathbb{Z}[\frac{1}{n}]$-module, define modular forms of wt k, level n, holo. at ∞, coefficients in K to be an element of $H^0(\overline{M_n}, \omega^{\otimes k} \underset{\mathbb{Z}[\frac{1}{n}]}{\otimes} K)$. At each cusp, it has q-expansion in $\mathbb{Z}[[q]] \underset{\mathbb{Z}}{\otimes} \mathbb{Z}[\frac{1}{n}, \zeta_n] \underset{\mathbb{Z}[\frac{1}{n}]}{\otimes} K$.

Thm. $n \geq 3$, K $\mathbb{Z}[\frac{1}{n}]$-module. $f \in H^0(\bar{M}_n, \omega^{\otimes k} \otimes_{\mathbb{Z}[\frac{1}{n}]} K)$. Suppose on each of the $\varphi(n)$ connected component of $\bar{M}_n \otimes_{\mathbb{Z}[\frac{1}{n}]} \mathbb{Z}[\frac{1}{n}, \zeta_n]$, there is at least one cusp at which the q-expansion of f is 0. Then $f = 0$.

Sketch: Reduce to the case K Artin local ring.

f induces zero section of $\omega^{\otimes k}$ over an open nbhd of at least one cusp on each connected component of $\bar{M}_n \otimes K \otimes \mathbb{Z}[\frac{1}{n}, \zeta_n]$, hence on an open dense in $\bar{M}_n \otimes K$.

$f \neq 0 \Rightarrow \exists$ nonempty closed subset Z containing no closed point of $\bar{M}_n \otimes K$, on which f is supported.

$\forall$ $z \in Z$ maximal ideal, $z \in \bar{M}_n \otimes K$ has depth zero $\Rightarrow$ closed point of $\bar{M}_n \otimes K$ $\Rightarrow$ contradiction. ///

Thm. $n \geq 3$, $k \geq 2$. $\forall$ $\mathbb{Z}[\frac{1}{n}]$-module K, the map

$$K \otimes H^0(\bar{M}_n, \omega^{\otimes k}) \longrightarrow H^0(\bar{M}_n, \omega^{\otimes k} \otimes_{\mathbb{Z}[\frac{1}{n}]} K)$$

is Bom.

Pf. Need to show $H^1(\bar{M}_n, \omega^{\otimes k}) = 0$.

$$\omega^{\otimes 2} \simeq \Omega^1_{\bar{M}_n / \mathbb{Z}[\frac{1}{n}]}(\log(\bar{M}_n - M_n)) \quad , \quad k \geq 2$$

$\Rightarrow$ $\omega^{\otimes k}$ has degree $> 2g-2$

$\overset{R\text{-}R}{\Rightarrow}$ $H^1(\bar{M}_n, \omega^{\otimes k}) = 0$. ///

p-adic modular forms

Hasse invariant A.

Let R be $\mathbb{F}_p$-algebra, E/R elliptic curve.

$F_{abs}: \mathcal{O}_E \to \mathcal{O}_E$ induces $F_{abs}^*: H^1(E, \mathcal{O}_E) \to H^1(E, \mathcal{O}_E)$ linear and

$F_{abs}^*(rx) = r^p F_{abs}^*(x)$, $\forall r \in R$.

ω base of $\omega_{E/R}$, dual base η of $H^1(E, \mathcal{O}_E)$, define $A(E,\omega) \in R$ by

$F_{abs}^*(\eta) = A(E,\omega)\cdot \eta$.

$\forall \lambda \in R^\times$, $\lambda^{-1}\eta$ dual to $\lambda\omega$. $F_{abs}^*(\lambda^{-1}\eta) = \lambda^{-p} F_{abs}^*(\eta) = \lambda^{-p} A(E,\omega)\eta = A(E,\lambda\omega)\cdot\lambda^{-1}\eta$

$\Rightarrow A(E,\lambda\omega) = \lambda^{1-p} A(E,\omega)$, $A \in M(\mathbb{F}_p, 1, p-1)$.

E/R elliptic curve, $H^1(E, \mathcal{O}_E)$ is the tangent space of E/R at origin, i.e.

the R-module of all translation-invariant derivations of E/R.

If $R/\mathbb{F}_p$, F_{abs}^* on $H^1(E, \mathcal{O}_E)$ is taking pth iterate.

On $Tate(8)$, $\omega_{can} = \frac{dt}{t}$ $\Rightarrow$ $H^1(Tate(8), \mathcal{O}_{Tate(8)}) = R\cdot D$, $D = t\frac{d}{dt}$.

$D(t) = t$ $\Rightarrow$ $D^p(t) = D(t)$ $\Rightarrow$ $D^p = D$ $\Rightarrow$ $F_{abs}^*\eta = \eta$ $\Rightarrow$ $A(Tate(8), \omega_{can}) = 1$

$\in \mathbb{F}_p[[8]]$.

Actually $A(E,\omega) \neq 0 \Leftrightarrow E$ p-ordinary.

$k \geq 4$ even, $E_k(z) = 1 - \frac{2k}{B_k} \sum \sigma_{k-1}(n) q^n$ defined over $\mathbb{Q}$.

$k = p-1$, $p \geq 5$ $\Rightarrow$ E_{p-1} defined over $\mathbb{Q} \cap \mathbb{Z}_p$ and after modulo p, $E_{p-1} = 1$.

Hence $A = E_{p-1} \bmod p$.

Let R_0 p-adic complete, $r \in R_0$.

$n \geq 1$, $(n, p) = 1$, $M(R_0, r, n, k)$ be the module of p-adic modular forms $/R_0$ of growth r, level n, wt k. $f \in M(R_0, r, n, k)$ assigns to each $(E/S, \alpha_n, Y)$ consisting of

(1) E/S elliptic curve, S/R_0, p nilpotent on S

(2) α_n level n structure

(3) Y section of $\omega^{\otimes 1-p}$ s.t. $Y \cdot E_{p-1} = r$.

a section $f(E/S, \alpha_n, Y) \in \Gamma(S, \omega_{E/S}^{\otimes k})$ depending only on $\simeq$ class, commutes with base change of S.

Equivalently $(E/R, \omega, \alpha_n, Y)$ level n structure

elliptic curve, base of $\omega_{E/R}$, $Y \in R$ s.t. $Y \cdot E_{p-1}(E, \omega) = r$.

$p \in R$ nilpotent

R R_0-algebra

$f(E/R, \omega, \alpha_n, Y) \in R$ depending only on $\simeq$ class, commuting with base change

and $f(E/R, \lambda\omega, \alpha_n, \lambda^{p-1} Y) = \lambda^{-k} f(E/R, \omega, \alpha_n, Y)$, $\lambda \in R^\times$.

Pass to limit, R can be taken to be p-adic complete R_0-algebra.

f is called holo. at ∞ if $\forall N \geq 1$, level n structure α_n

$f(\mathrm{Tate}(q^n), \omega_{can}, \alpha_n, r \cdot E_{p-1}(\mathrm{Tate}(q^n), \omega_{can})^{-1}) \in \mathbb{Z}[\![q]\!] \otimes R_0/p^N R_0[\zeta_n]$

$S(R_0, r, n, k) \subset M(R_0, r, n, k)$ all forms holo. at ∞.

$$\overset{M}{S}(R_0, r, n, k) = \varprojlim \overset{M}{S}(R_0/p^N, r, n, k).$$

Determination of $M(R_0, r, n, k)$ and $S(R_0, r, n, k)$.

$p \geq 5$, $n \geq 3$, $\mathcal{L} = \omega^{\otimes 1-p}$, p nilpotent in R_0.

Consider functors $F_{R_0, r, n} : S \longrightarrow \{(E/S, \alpha_n, Y)\}/\simeq_S$

$\wr\wr$

$F_{R_0, r, n} : S \longrightarrow \left\{ \begin{array}{l} R_0\text{-morphism } g : S \to M_n \otimes R_0,\ Y \text{ section of} \\ g^*(\mathcal{L}) \text{ s.t. } Y \cdot g^*(E_{p-1}) = r \end{array} \right\}$

$\cap$

$F_{R_0, n} : S \longrightarrow \{ R_0\text{-morphism } g : S \to M_n \otimes R_0,\ Y \text{ section of } g^*\mathcal{L} \}$

$\triangle$

this is rep'd by $\underline{\mathrm{Spec}}_{M_n \otimes R_0}(\mathrm{Sym}\, \check{\mathcal{L}}) \longrightarrow M_n \otimes R_0$

$\Rightarrow F_{R_0, r, n}$ is rep'd by closed subscheme cut out by $E_{p-1} - r$

$$M(R_0, r, n, k) = H^0(\underline{\mathrm{Spec}}_{M_n \otimes R_0}(\mathrm{Sym}\, \check{\mathcal{L}}(E_{p-1} - r)), \omega^{\otimes k})$$

$$= H^0(M_n \otimes R_0, (\bigoplus_{j \geq 0} \omega^{\otimes k + j(p-1)})/(E_{p-1} - r))$$

$$= H^0(M_n \otimes R_0, \bigoplus_{j \geq 0} \omega^{\otimes k + j(p-1)})/(E_{p-1} - r)$$

$$= \bigoplus_{j \geq 0} M(R_0, n, k + j(p-1))/(E_{p-1} - r).$$

Moreover, $S(R_0, r, n, k) = H^0(\underline{\mathrm{Spec}}_{\overline{M}_n \otimes R_0} (\mathrm{Sym}\, \check{\mathcal{L}}) / (E_{p-1} - r), \omega^{\otimes k})$

$= H^0(\overline{M}_n \otimes R_0, \bigoplus_{j \geq 0} \omega^{k+j(p-1)} / (E_{p-1} - r))$.

Thm.
Let $p \geq 5$, $n \geq 3$, $k \geq 2$, R_0 p-adic complete, $r \in R_0$ not zerodivisor. Then

$$\varprojlim H^0(\overline{M}_n, \bigoplus_{j \geq 0} \omega^{k+j(p-1)}) \otimes_{\mathbb{Z}[\frac{1}{n}]} (R_0/p^N) / (E_{p-1} - r) \to \varprojlim S(R_0/p^N, r, n, k)$$

is an $\simeq$.

Pf. Let $\mathcal{S} = \bigoplus_{j \geq 0} \omega^{k+j(p-1)} \in \mathrm{QCoh}(\overline{M}_n)$, $\mathcal{S}_N = \mathcal{S} \otimes R_0/p^N$

$$0 \to \mathcal{S}_N \xrightarrow{E_{p-1} - r} \mathcal{S}_N \to \mathcal{S}_N / (E_{p-1} - r) \to 0$$

$$\Rightarrow 0 \to H^0(\overline{M}_n, \mathcal{S}_N) \xrightarrow{E_{p-1} - r} H^0(\overline{M}_n, \mathcal{S}_N) \to H^0(\overline{M}_n, \mathcal{S}_N / (E_{p-1} - r))$$
$$\to H^1(\overline{M}_n, \mathcal{S}_N) \xrightarrow{E_{p-1} - r} H^1(\overline{M}_n, \mathcal{S}_N) \to H^1(\overline{M}_n, \mathcal{S}_N / (E_{p-1} - r)) \to \cdots$$

Base Change Theorem $\Rightarrow$ $H^0(\overline{M}_n, \mathcal{S}_N) = H^0(\overline{M}_n, \mathcal{S}) \otimes R_0/p^N$, $H^1(\overline{M}_n, \mathcal{S}_N) = 0$

Then take inverse lim. ///

Lemma. $p \geq 5$, $n \geq 3$, $k \geq 2$. $\forall j \geq 0$, $\underbrace{H^0(\overline{M}_n \otimes \mathbb{Z}_p, \omega^{\otimes k + j(p-1)})}_{H_j} \xrightarrow{E_{p-1}} H_{j+1}$ has a section.

Pf. The cokernel is finite free $\mathbb{Z}_p$-module. ///

Choose once and for all sections and denote the image by $B(n, k, j+1)$.

$$\Rightarrow H^0(\overline{M}_n, \omega^{\otimes k+(j+1)(p-1)}) \simeq E_{p-1} H^0(\overline{M}_n, \omega^{\otimes k+j(p-1)}) \oplus B(n, k, j+1)$$

$H^0(\overline{M}_n, \omega^{\otimes k}) = B(n, k, 0)$, $B(R_0, n, k, j) = B(n, k, j) \otimes_{\mathbb{Z}_p} R_0$

$$\Rightarrow \bigoplus_{a=0}^{j} B(R_0, n, k, a) \xrightarrow{\sim} S(R_0, n, k+j(p-1)) \qquad (*)$$

$$(b_a) \longmapsto \sum E_{p-1}^{j-a} b_a$$

Let $B^{rigid}(R_0, r, n, k)$ be R_0-module of all formal sums $\sum_{a=0}^{\infty} b_a$, $b_a \in B(R_0, n, k, a)$

such that $\forall N > 0$, $\exists M > 0$, $\forall a \geq M$, $b_a \in p^N B(R_0, n, k, a)$

Prop. p, n, k as before. The inclusion $B^{rigid}(R_0, r, n, k)$ in the p-adic completion of $H^0(\overline{M}_n, \bigoplus_{j \geq 0} \omega^{k+j(p-1)})$ induces via $(*)$ above an isom.

$$\Phi : B^{rigid}(R_0, r, n, k) \xrightarrow{\sim} S(R_0, r, n, k)$$

$$\sum b_a \longmapsto \sum \frac{r^a b_a}{E_{p-1}^a} = \Phi(\sum b_a)$$

where $\Phi(\sum b_a)(E/S, \alpha_n, Y) = \sum b_a(E/S, \alpha_n) \cdot Y^a$

Formal Schemes.

$n \geq 3$, $p \nmid n$, R_0 p-adic complete, $r \in R_0$.

$M_n(R_0, r)$ ($\overline{M}_n(R_0, r)$) the formal scheme over R_0 given the compatible family of R_0/p^N-schemes $\underline{\mathrm{Spec}}_{M_n \otimes R_0/p^N}(\mathrm{Sym}\,\check{\underline{L}} / (E_{p-1} - r))$. Then

$M(R_0, r, n, k) = H^0(M_n(R_0, r), \omega^{\otimes k})$

$S(R_0, r, n, k) = H^0(\overline{M}_n(R_0, r), \omega^{\otimes k})$