

Igusa tower : $(\sim) W^{K^{un}} \simeq (\mathcal{O}_{Ig} \otimes \Lambda)^{\mathbb{Z}_p^*}$

$$N \geq 3, (N, p) = 1.$$

$$Ig_n : R \in \text{Nilp}_{\mathbb{Z}_p}^{op} \rightarrow (E/R, \varphi_n, \varphi_{p^n}) / \simeq$$

$$\varphi_{p^n} : (E(\mathbb{P}^n)^0)^\circ \rightarrow \mu_{p^n}.$$

Ig_n is representable by a formal affine scheme.

Let $n \rightarrow \infty$.

$$Ig : R \in \text{Nilp}_{\mathbb{Z}_p}^{op} \rightarrow (E/R, \varphi_n, \varphi_{p^\infty}) / \simeq$$

$$\varphi_{p^\infty} : \varinjlim E(\mathbb{P}^\infty)^0 \rightarrow \mu_{p^\infty} \simeq \widehat{G}_m$$

$$\underline{\underline{\mu_{\text{pro}}(R)}} = (\underline{\underline{1 + \text{Nilp}(R)}}, x) = \widehat{G}_m(R).$$

x s.t. $(x-1)^{p^n} = 0$ for some n .

$$p^N = 0 \text{ in } R \quad \Downarrow \quad (x-1)^m = 0 \text{ for some } m.$$

I_g is also represented by an affine formal scheme, pro-étale \mathbb{Z}_p^X -torsor over $X^{\text{ord}}(N)$.

$$\mathbb{Z}_p^X = \text{Aut}(\mu_{\text{pro}}) = \text{Aut}(\mathbb{Z}_p)$$

acts on φ_p .

$$f \in H^0(X^{\text{ord}}(N), \omega^k)$$

$$P: I_g \rightarrow X^{\text{ord}}(N)$$

$$\begin{array}{ccc} \leadsto H^0(\mathbb{I}_g, p^* \omega^k) \ni p^* f & & \\ & \nearrow \text{is} & \downarrow \\ \ell_{p^*} \text{ induces} & \mathcal{O}_{\mathbb{I}_g} \left(\frac{dt}{t} \right)^k \ni \tilde{f} & \end{array}$$

a trivialization of invariant differentials
by pulling back for $(\hat{G}_m, \frac{dt}{t})$.

Weights translates into a character
of \mathbb{Z}_p^\times .

$$a \in \mathbb{Z}_p^\times \curvearrowright \hat{G}_m$$

$$\underline{t} \mapsto \underline{t^a} \quad \underbrace{\text{Lie}}_{\sim} \quad a: \text{Lie } \hat{G}_m \rightarrow \text{Lie } \hat{G}_m$$

dualizing \leadsto $a : \frac{dt}{t} \mapsto a \frac{dt}{t}$

$$a \cdot \hat{f} = a^{-k} \tilde{f}$$

$$\chi : \mathbb{Z}_p^\times \rightarrow \mathbb{Z}_p^\times \quad \text{a character}$$

$$S_\chi^{\text{ord}}(N) := \{ f \in \mathcal{O}_{\mathbb{Z}_p} \mid \bar{a} \cdot f = \chi(a) f \}$$

Serre's duality,

$$X/\mathbb{Z}_p(N) \rightsquigarrow H^1(X, \Omega_{X/\mathbb{Z}_p}) \rightarrow \mathbb{Z}_p$$

Want to construct

$$\langle, \rangle : H^0(X^{\text{ord}}, \omega^{\text{Kun}}) \times H_c^1(X^{\text{ord}}, \omega^{\text{Kun}(-1)})$$

$$\rightarrow H_c^1(X^{\text{ord}}, \omega^2(-D) \otimes_{\mathbb{Z}_p} \Lambda) \xrightarrow{\text{red}} \Lambda.$$

$$\xrightarrow{\text{red}}: H_c^1(X^{\text{ord}}, \omega^2(-D) \otimes_{\mathbb{Z}_p} \Lambda)$$

$$\rightarrow H^1(X, \omega^2(-D) \otimes \Lambda) \rightarrow H^1(X, \omega^2(-D)) \otimes \Lambda$$

$$\xrightarrow{\text{KS} \otimes 1} H^1(X, \Omega_X / \mathbb{Z}_p) \otimes \Lambda \xrightarrow{\text{res}} \Lambda.$$

$$f \in H^0(X^{\text{ord}}, \omega^{k_{\text{un}}}) \Leftrightarrow \mathcal{O}_{X^{\text{ord}}} \xrightarrow{f} \omega^{k_{\text{un}}}.$$

$$\omega^{2-k_{\text{un}}}(-D) := \omega^2(-D) \otimes \underline{\text{Hom}}(\omega^{k_{\text{un}}}, \Lambda \otimes \mathcal{O}_{X^{\text{ord}}})$$

$$f \text{ induces } \omega^{2-k_{\text{un}}}(-D) \rightarrow \omega^2(-D) \otimes \Lambda \rightarrow$$

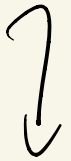
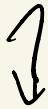
$$H_c^1(X^{\text{ord}}, \omega^{2-k_{\text{un}}}(-D)) \rightarrow H_c^1(X^{\text{ord}}, \omega^2(-D) \otimes \Lambda).$$

Prop. 4.17. $(f, g) \in H^0(X^{\text{ord}}, \omega^{k_{\text{un}}}) \times H_c^1(X^{\text{ord}}, \omega^{2-k_{\text{un}}}(-D))$, we have

$$\langle U_p f, g \rangle = \langle f, Fg \rangle.$$

Proof. Step 1.

$$H^0(X^{\text{ord}}, \omega^{k_{\text{un}}}) \times H_c^1(X^{\text{ord}}, \omega^{2-k_{\text{un}}}(-D)) \rightarrow \Delta$$



$$\prod_{k \in \mathbb{Z}} H^0(X^{\text{ord}}, \omega^k) \times H_c^1(X^{\text{ord}}, \omega^{2-k}(-D)) \rightarrow \prod_{k \in \mathbb{Z}} \mathbb{Z}_p$$

Step 2: Prove $\langle U_p f, g \rangle = \langle f, Fg \rangle$

for every $k \in \mathbb{Z}$.

Work with X_n^{ord} . By defn of

$$H_c^1(X_n^{\text{ord}}, \mathcal{F}) := \varprojlim H^1(X_n, \mathcal{I}^n \mathcal{F})$$

On H_c^1 , F acts as

$$F: P_2^* \mathcal{I}^{m \times} \omega^{2k}(-D) \rightarrow P_1^! \mathcal{I}^{-l+s} \omega^{2k}(-D)$$

$\begin{matrix} s \\ \downarrow \\ \times \end{matrix}$

Apply $R\text{Hom}(\ , \omega_{X_0(p)/\mathbb{Z}_p}) =: D$

$$D(F): P_1^* \mathcal{I}^{-s+l} \omega^k \rightarrow P_2^! \mathcal{I}^{-ms} \omega^k$$

Lemma 4.14? $D(F) = U_p$.

\uparrow ?

prop 3.6?

$$\Omega_{X/\mathbb{Z}_p}(\log(ss+D))$$

$$\uparrow \\ P(T_p) = T_p$$

$$\cong X_{0(p)}/\mathbb{Z}_p (\log D)^{-1}$$

$$\cong W_{X_{0(p)}/\mathbb{Z}_p}$$

Let $s \rightarrow \infty$, take H^0 .

$$= \text{codim } H^0(\mathcal{I}_{j \neq p}^{-s})$$

Prop 4.18. Restrict $\langle \cdot, \cdot \rangle$ to

$$e(U_p) H^0(\mathcal{X}^{\text{ord}}, W^{K^{\text{un}}}) \times e(F) H_c^1(\mathcal{X}^{\text{ord}}, W^{\text{un}}(D)).$$

(1) $\langle \cdot, \cdot \rangle$ becomes a perfect pairing.

(2) $\langle U_p f, g \rangle = \langle f, Fg \rangle.$

(3) Have commutative diagram

$$e(U_p) H^0(\mathbb{A}^{\text{ord}}, \omega^k) \times e(F) H_c^1(\mathbb{A}^{\text{ord}}, \omega^{\frac{k}{2}}) \rightarrow \mathbb{Z}_p$$

$$i \uparrow$$

$$\cong$$

$$\downarrow j$$

$$e(T_p) H^0(X, \omega^k) \times e(T_p) H^1(X, \omega^{\frac{k}{2}}(\rightarrow))$$

Serre's duality.

Proof. (2) follows from 4.17.

(1) follows from (3): i & j are

isom for $k \geq 3$. & the same trick

as step 1 in 4.17.

Prove (3) for $k \geq 3$.

Observe that.

$$H^0(\mathcal{X}^{\text{ord}}, \omega^k) \times H_c^1(\mathcal{X}^{\text{ord}}, \omega^{2k}(-D)) \longrightarrow \mathbb{Z}_p$$

$$\begin{array}{ccc} & & \nearrow \\ i \uparrow & \circlearrowleft & \downarrow j \\ H^0(X, \omega^k) \times H^1(X, \omega^{2k}(-D)) & & \end{array}$$

Commutates.

For any $f \in e(T_p)H^0(X, \omega^k)$ and $g \in e(F)H_c^1(\mathcal{X}^{\text{ord}}, \omega^{2k}(-D))$.

$$\begin{aligned} \langle e(U_p)z(f), g \rangle &= \langle z(f), e(F)g \rangle \\ &= \langle z(f), g \rangle. \end{aligned}$$

$$\langle f, e(T_p)j(g) \rangle = \langle e(T_p)f, j(g) \rangle$$

$$= \langle f, j(g) \rangle.$$

$$\approx \langle \text{if}_1, g \rangle.$$

□.