GAUGE THEORY AND TOPOLOGY: AN INTRODUCTION PRELIMINARY SYLLABUS

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Contents. The goal of the course is to explore the interplay between topology and gauge theory, starting from the Dirac monopole (1931) up to the advent of Seiberg-Witten theory in low-dimensional topology (1994). The course will be roughly divided in following three parts (each consisting of 6-8 hours):

1. *The equations of gauge theory.* Connections on principal bundles and their curvature, Chern-Weil and Chern-Simons theory, the Yang-Mills(-Higgs) functional, Bogomolnyi bounds, ASD equations and the BPST instanton, BPS monopoles.
2. *Analytic construction of moduli spaces.* Foundations of differential topology in infinite dimensions: Sobolev spaces, Fredholm operators, elliptic regularity, Hodge theory. Case study: the vortex equations and symmetric products of a surface.
3. *Spin geometry and Seiberg-Witten theory.* Clifford algebras, spin geometry and Dirac operators. Seiberg-Witten equations and construction of the invariants, adjunction inequalities and proof of Thom conjecture, invariants of symplectic manifolds.

Along the way, we will explain why the solutions to the Seiberg-Witten equations are called monopoles, and try to discuss some of the physical interpretations behind the objects and constructions we consider.

References. I plan to write notes for the lectures. Some books we will follow are:

- Roe - Elliptic operators, asymptotic methods and topology;
- Donaldson, Kronheimer - The geometry of 4-manifolds;
- Morgan - The Seiberg-Witten Equations and Applications to the Topology of Smooth Four-Manifolds;
- Manton, Sutcliffe - Topological Solitons.
- Hamilton - Mathematical gauge theory.

We will also refer to papers in the literature as the course progresses, for example:

- Garcia-Prada - A direct existence proof for the vortex equations over a compact Riemann surface;
- Kronheimer, Mrowka - The genus of embedded surfaces in the projective plane;
- Taubes - The Seiberg-Witten invariants and symplectic forms.

Prerequisites. First graduate level courses in analysis, differential geometry, differential and algebraic topology. Basic notions of classical electrodynamics will be helpful for context.

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