Speaker: Lev Borisov

Title: Modular curves $X_1(n)$ as moduli of point arrangements

Abstract: For a complex elliptic curve E and a point p of order n on it, the images of the points $p_k = kp$ under the Weierstrass embedding of E into CP^2 are collinear if and only if the sum of indices is divisible by n. We prove that for n at least 10 a collection of n points in P^2 with these properties comes (generically) from a point of order n on an elliptic curve. In the process, we discover amusing identities between logarithmic derivatives of the theta function at rational points. I will also discuss potential applications of these results to bounds on the numbers of Hecke eigenforms for $\Gamma_1(n)$ of positive analytic rank, although this is rather speculative. This is joint work with Xavier Roulleau, see: https://arxiv.org/pdf/2404.04364.