EXERCISES #3

DOT PRODUCT

Exercise 1. What is the angle between $\vec{i} + \sqrt{3}\vec{j}$ and the positive x-direction?

Exercise 2. Find $\vec{u} \cdot \vec{v}$ for the following.

(1) $\vec{u} = \langle 1, 3, -5 \rangle, \vec{v} = \langle 4, 3, 19 \rangle$
(2) $\vec{u} = \langle 0, 4 \rangle, \vec{v} = \langle 8, -6 \rangle$
(3) $\vec{u} = \langle 7, 11 \rangle, \vec{v} = \langle 9, 2 \rangle$
(4) $\vec{u} = 2\vec{i} + \vec{k}, \vec{v} = \vec{i} - 6\vec{j}$

Exercise 3. Show that $\vec{i} \cdot \vec{j} = \vec{j} \cdot \vec{k} = \vec{k} \cdot \vec{i} = 0$ and $\vec{i} \cdot \vec{i} = \vec{j} \cdot \vec{j} = \vec{k} \cdot \vec{k} = 1$.

Exercise 4. Let θ be the angle between $\vec{u} = \langle 5, 1 \rangle$ and $\vec{v} = \langle 3, 2 \rangle$. What is $\cos\theta$?

Exercise 5. Let θ be the angle between $\vec{u} = \vec{i} - 4\vec{j} + \vec{k}$ and $\vec{v} = -3\vec{i} + \vec{j} + 5\vec{k}$. What is $\cos\theta$?

Exercise 6. Determine whether the triangle with vertices $P = (1, -3, -2)$, $Q = (2, 0, -4)$, $R = (6, -2, -5)$ is right-angled.

Exercise 7. Find the values of x such that the angle between the vectors $\langle 2, 1, -1 \rangle$ and $\langle 1, x, 0 \rangle$ is $\frac{\pi}{4} = 45°$.

Exercise 8. Find the $\vec{v}$-direction component of $\vec{u}$, where $\vec{u} = \langle 3, -1, 1 \rangle$ and $\vec{v} = \langle 4, 7, -4 \rangle$.

Exercise 9. Find the projection of $\vec{u} = 5\vec{j} - \vec{k}$ to $\vec{v} = 2\vec{i} + \vec{j} + 3\vec{k}$.

Exercise 10. Show that, for any vectors $\vec{u}$ and $\vec{v}$ (in either 2D or 3D), $\vec{v} - \text{proj}_{\vec{u}}\,\vec{v}$ is orthogonal to $\vec{u}$.