EXERCISES #6

PLANES AND SURFACES

Exercise 1. True or False:

(1) Two planes parallel to a third plane are parallel.
(2) Two planes orthogonal to a third plane are parallel.
(3) Two planes parallel to a line are parallel.
(4) Two planes either intersect or are parallel.
(5) A plane and a line either intersect or are parallel.
(6) Two planes orthogonal to a line are parallel.
(7) Two lines parallel to a plane are parallel.
(8) Two lines orthogonal to a plane are parallel.

Exercise 2. Find an equation of the following planes.

(1) The plane that passes through the point $A = (1, 0, 4)$ and is orthogonal to the vector $\vec{n} = \langle 3, 2, -1 \rangle$.
(2) The plane that passes through the point $A = (-5, 2, 1)$ and is parallel to the vectors $\vec{u} = \langle 1, -2, 2 \rangle$ and $\vec{v} = \langle 4, -1, -2 \rangle$.
(3) The plane that passes through the points $A = (2, 4, 5)$, $B = (1, 0, 3)$ and $C = (5, 8, 3)$.
(4) The plane that passes through the points $A = (2, 0, -4)$, $B = (0, 1, 4)$ and $C = (5, 2, 5)$.

Exercise 3. Determine whether the given objects intersect or not. Determine whether the given objects are parallel or not.

(1) The line $L_1 : x = t, y = 6 - 2t, z = -7 + t$ and the plane $P_1 : 2x + 3y + z = -1$.
(2) The line L_1 passing through $A = (2, -1, 5)$ and $B = (0, -7, 9)$, and the plane $P_1 : 5x - 3x - 2z = 2$.
(3) The plane $P_1 : 2x + 5y + z = 3$ and the plane $P_2 : 6x + 15y + 3z = 2$.
(4) The plane $P_1 : -x + 2y + 3z = 0$ and the plane $P_2 : 3x + 2y - z = 3$.

Exercise 4. Find an equation of the following lines.

(1) The intersection between the planes $P_1 : 3x - y + 2z = 2$ and $P_2 : -2x + y + z = 0$.
(2) The intersection between the plane $P_1 : 7x - 3y + 4z = -7$ and the plane P_2 that passes through $(4, 4, 0)$ and is orthogonal to the vector $\vec{n} = \langle 1, -1, 1 \rangle$.

Exercise 5. Find the angle between the following objects.

(1) The line $L_1 : x = t, y = 2 - 2t, z = 6 + 2t$ and the plane $P_1 : x - y = 0$.
(2) The line $L_1 : x = 94 + t, y = e^{150\pi} - t, z = 0.01524$ and the plane
$$P_1 : (1 + 3\sqrt{3})x + (1 - 3\sqrt{3})y - 4z = e^{2022^{e^{2022}}}.$$
(3) The plane P_1 that passes through $A = (1, -1, 1)$, $B = (\frac{1}{2}, 2, 2)$ and $C = (2, 5, 7)$ and the plane $P_2 : (2 + 6\sqrt{3})x + (3 + 2\sqrt{3})y + (6 - 3\sqrt{3})z = 0$.