EXERCISES #8

DERIVATIVES AND INTEGRALS OF VECTOR FUNCTIONS

Exercise 1. Find the domain of the vector function and determine whether they are closed and/or bounded. Determine whether the domain is compact or not.

(1) $\vec{r}(t) = \langle \sqrt{t}, t^2 + 1, \ln(t) \rangle$
(2) $\vec{r}(t) = \langle \sin(e^t), \sqrt{e^t}, \sqrt{-t} \rangle$
(3) $\vec{r}(t) = \langle \sqrt{2+t}, \sqrt{2-t}, \frac{1}{t+4} \rangle$

Exercise 2. Find the limit, or explain why it does not exist.

(1) $\lim_{t\to 0} \langle t^2 + 1, \frac{1}{t+1}, e^t \rangle$.
(2) $\lim_{t\to 0} \langle \frac{\sin(t)}{t}, \frac{e^t - 1}{t}, t \rangle$.
(3) $\lim_{t\to \infty} \langle \frac{1}{t}, t+1, e^t \rangle$.
(4) $\lim_{t\to \infty} \langle \frac{t^2+1}{2t^2+t}, \frac{\ln(t)}{t}, \frac{t+1}{t^2+1} \rangle$.
(5) $\lim_{t\to 1} \langle \frac{t-1}{t^2-1}, \frac{e^t-1}{t}, \frac{1}{t+3} \rangle$.

Exercise 3. Find $\vec{r}\,'(t)$.

(1) $\vec{r}(t) = \langle t^5, -2t \rangle$
(2) $\vec{r}(t) = e^{3t}\vec{i} + \cos(t)\vec{j} + \frac{1}{1+t}\vec{k}$
(3) $\vec{r}(t) = \langle \sqrt{t+2}, 2, \frac{1}{t^3} \rangle$

Exercise 4. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter t.

(1) $\vec{r}(t) = \langle \sin t, \cos 2t, \cos t + \sin t \rangle, \quad t = 0.$
(2) $\vec{r}(t) = \langle e^{-t}, t^2 + 2t, e^t \rangle, \quad t = 0.$
(3) $\vec{r}(t) = \langle \ln(t+1), \cos(t), \frac{1}{t^2-3} \rangle, \quad t = 0.$

Exercise 5. Find the unit tangent vector $\vec{T}(t)$ at the given point.

(1) $\vec{r}(t) = \langle \frac{1}{t+1}, \frac{1}{t^2+1}, \sin^2 t \rangle, \quad (1, 1, 0)$
(2) $\vec{r}(t) = \langle \sin t, 5t, \cos t \rangle, \quad (0, 0, 1)$
(3) $\vec{r}(t) = \langle (t+1)^{2/3}, e^{t-7}, \frac{7}{t} \rangle, \quad (4, 1, 1)$

Exercise 6. Find a vector equation for the tangent line to the curve

$$x = \sqrt{t^2+3}, \quad y = \ln(t^2+3), \quad z = t,$$

at $(2, \ln 4, 1)$.

Exercise 7. Evaluate the integral.

(1) $\int_1^3 (t\vec{i} - 2t\vec{j} - (4t^3+3)\vec{k})dt$.
(2) $\int \left(\frac{1}{1+t}\vec{i} + t^{2/3}\vec{j} + \sin(t)\vec{k} \right) dt$.
(3) $\int_0^{\pi/4} (\cos(2t)\vec{i} + e^t\vec{j} + \sin(t)\vec{k})dt$.

Exercise 8. Find $\vec{r}(t)$.

(1) $\vec{r}\,'(t) = 2t\vec{i} + e^t\vec{j} + \sqrt{t}\vec{k}, \quad \vec{r}(1) = \vec{i} + \vec{j} - \vec{k}.$
(2) $\vec{r}\,'(t) = \frac{1}{t+2}\vec{i} + (t+1)^2\vec{j} + e^{-2t}\vec{k}, \quad \vec{r}(0) = 3\vec{i} - \vec{k}.$
(3) $\vec{r}\,'(t) = \langle t + e^{-t}, 3t^2 - 2, \sin(t) - e^t \rangle, \quad \vec{r}(0) = \langle 3, -2, 1 \rangle.$

Exercise 9. If $\vec{r}(t)$ is some curve on the sphere $x^2 + y^2 + z^2 = 1$ (namely if $|\vec{r}(t)| = 1$ regardless of t), then show that $\vec{r}\,'(t)$ is always orthogonal to $\vec{r}(t)$.