EXERCISES #9

CALCULUS FOR CURVES AND MOTIONS

Exercise 1. Find the arclength of the curve segment.

(1) $\vec{r}(t) = \langle t, 3\cos t, 3\sin t\rangle$, $-5 \le t \le 5$.
(2) $\vec{r}(t) = \langle t, t^2, \frac{2t^3}{3}\rangle$, $0 \le t \le 1$.
(3) $\vec{r}(t) = \langle t^2, 9t, 4t^{3/2}\rangle$, $1 \le t \le 4$.

Exercise 2. Find the arclength parametrization.

(1) $\vec{r}(t) = \langle \cos(t^3), \sin(t^3)\rangle$, starting from $t = 0$, to the direction of increasing t.
(2) $\vec{r}(t) = \langle \cos(t^4), \sin(t^4)\rangle$, starting from $t = 0$, to the direction of decreasing t.
(3) $\vec{r}(t) = \langle 5 - t, 4t - 3, 3t\rangle$, starting from $(5, -3, 0)$, to the direction of increasing t.
(4) $\vec{r}(t) = \langle e^t \sin t, e^t \cos t, \sqrt{2}e^t\rangle$, starting from $(0, -e^\pi, \sqrt{2}e^\pi)$, to the direction of decreasing t.
(5) $\vec{r}(t) = \langle e^t \sin t, e^t \cos t, \sqrt{2}e^t\rangle$, starting from $(0, -e^\pi, \sqrt{2}e^\pi)$, to the direction of increasing t.

Exercise 3. Explain why the parametrization of $\vec{r}(t) = \langle \cos(t^2), \sin(t^2)\rangle$ with respect to arclength, starting from $t = -3$, to the positive direction, does not exist.

Exercise 4. An object moves with position function $\vec{r}(t) = \langle t^2, e^t \sin t, e^t \cos t\rangle$. Find the velocity and acceleration $\vec{v}(t)$ and $\vec{a}(t)$.