

EXERCISES #11

LIMITS AND CONTINUITY IN SEVERAL VARIABLES

Exercise 1. Find the limit.

(1)

$$\lim_{(x,y) \rightarrow (2,1)} (x^3y - 3y^2)$$

(2)

$$\lim_{(x,y) \rightarrow (0,-10)} (x^4y^2 + 3xy - 2y + 4)$$

(3)

$$\lim_{(x,y) \rightarrow (2,2)} \frac{\sin(x-y)}{x-y}$$

(4)

$$\lim_{(x,y) \rightarrow (1,-1)} \sin\left(\frac{e^{(x+y)^2} - 1}{x+y}\right)$$

(5)

$$\lim_{(x,y) \rightarrow (1,1)} \left(\frac{x^2y^3 - x^3y^2}{x^2 - y^2}\right)$$

(Hint: $x^2y^3 - x^3y^2 = x^2y^2(y-x)$, and $x^2 - y^2 = (x-y)(x+y)$)

(6)

$$\lim_{(x,y) \rightarrow (0,0)} xy \sin\left(\frac{1}{x^2 + y^2}\right)$$

(7)

$$\lim_{(x,y) \rightarrow (0,0)} \left(\frac{xy^2 \sin(x)}{x^2 + y^2}\right)$$

(8)

$$\lim_{(x,y) \rightarrow (0,0)} \left(1 + \frac{x^3 \sin(x)}{x^2 + y^4}\right)$$

Exercise 2. Show that the limit does not exist.

(1)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x}{2x + y}$$

(2)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy}{x^2 + 3y^2}$$

(3)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2y^3}{x^4 + y^6}$$

(4)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^4 - 2x^2 y^2 - y^4}$$

(5)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 y}{x^5 + 3x^2 y^2 - y^3}$$

(6)

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{xy^2 + yz^2 + zx^2}{x^3 + y^3 + z^3}$$

(7)

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^3 y^2 z}{x^6 + x^3 y^3 + y^6 + z^6}$$

(8)

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2 y + yz^2}{x^5 + x^3 y + x^2 z^2 + xy^2 z}$$

Exercise 3. Find the limit, if it exists, or show that the limit does not exist.

(1)

$$\lim_{(x,y) \rightarrow (1,2)} \frac{2x - y}{4x^2 - y^2}$$

(2)

$$\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^4 + y^2 + z^3}{x^4 + 2y^2 + z}$$

(3)

$$\lim_{(x,y) \rightarrow (1,1)} \frac{y - x}{1 - y + \ln x}$$

(4)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{y^2 \sin^2 x}{x^4 + y^4}$$