EXERCISES #13

TANGENT PLANES AND APPROXIMATIONS

Exercise 1. Find an equation of the tangent plane of the given surface at the specified point.

(1) The graph of $f(x, y) = x^2 + y^2 + x - 2y$, at $x = 0, y = 1$.
(2) The graph of $f(x, y) = y^2 e^x$, at $(0, 3, 9)$.
(3) The graph of $f(x, y) = \frac{x}{y^2}$, at $(-4, 2, -1)$.
(4) The graph of $f(x, y) = x \sin(x + y)$, at $(-1, 1, 0)$.
(5) The surface $x^2 + 4y^2 + 9z^2 = 1$, at $(\frac{3}{7}, \frac{1}{7}, \frac{2}{7})$.
(6) The surface $xy + yz + zx = 0$, at $(-1, 2, 2)$.
(7) The surface $xe^z + yz = xy$, at $(\frac{1}{2}, -e, 1)$.

Exercise 2. Approximate the number.

(1) $f(0.01, 0.01)$, for $f(x, y) = \sin(x + 2y)$.
(2) $f(0.1, -0.1)$, for $f(x, y) = e^{y \cos(x)}$.
(3) $f(\frac{\pi}{2} + 0.02, 0.01)$, for $f(x, y) = xy \sin(x + y)$.
(4) $f(3.02, 1.97, 5.99)$, for $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$.
(5) $f(1.02, 0.01, 1.02)$, for $f(x, y, z) = z^2 \ln(x^2 - y^2)$.

Exercise 3. Find the differential Δf.

(1) $f(x, y) = x^2 - xy + 3y^2$.
(2) $f(x, y) = 1 + x \ln(xy - 5)$.
(3) $f(x, y, z) = xze^{-y^2 - z^2}$.

Exercise 4. Find the margin of error of the quantity.

(1) $f(x, y) = xe^{xy}$, with $x = 2 \pm 0.1$ and $y = 0 \pm 0.2$.
(2) $f(x, y) = x \ln(y + 1)$, with $x = -1 \pm 0.1$ and $y = 0 \pm 0.1$.
(3) The volume of the box with sides $(2 \pm 0.05) \times (10 \pm 0.07) \times (10 \pm 0.02)$.
(4) $f(x, y, z) = x^2 y^4 + y^3 z^5$, with $x = 3 \pm 0.01$, $y = 2 \pm 0.01$ and $z = 1 \pm 0.01$.

Exercise 5. Approximate using the table of values.

(1) The wave heights, when the wind has been blowing for 32 hours at 43 knots. The following is the table of wave heights in the open sea, measured in feet.

Duration (hours)

t \ v	5	10	15	20	30	40	50
Wind 20	5	7	8	8	9	9	9
speed 30	9	13	16	17	18	19	19
(knots) 40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

1

(2) The perceived temperature, when the actual temperature is $-17°$C and the wind speed is 52km/h. The following is the table of perceived temperature, measured in °C.

Wind speed (km/h)

	v 20	30	40	50	60	70
T						
Actual temperature (°C) -10	-18	-20	-21	-22	-23	-23
-15	-24	-26	-27	-29	-30	-30
-20	-30	-33	-34	-35	-36	-37
-25	-37	-39	-41	-42	-43	-44

(3) The surface area of a human body, when the weight is 177 pounds and the height is 5.7 feet. The following is the table of the surface area of a human body, measured in square feet.

Weight (pounds)

w h	150	160	170	180	190	200
Height (feet) 5	17.9	18.4	18.8	19.3	19.7	20.2
5.5	19.1	19.7	20.2	20.7	21.2	21.6
6	20.4	20.9	21.5	22	22.5	23
6.5	21.6	22.2	22.8	23.3	23.9	24.4