EXERCISES #14

CHAIN RULE

Exercise 1. Find the derivative(s).

(1) $\frac{d}{dt}f(x(t),y(t))$, where
$$f(x,y)=xy^2,\quad x(t)=t^3,\quad y(t)=\frac{1}{t}$$

(2) $\frac{d}{dt}f(x(t),y(t))$, where
$$f(x,y)=e^{xy},\quad x(t)=\ln(t),\quad y(t)=t$$

(3) $\frac{d}{dt}f(x(t),y(t))$, where
$$f(x,y)=x^3+xy^2,\quad x(t)=\sin(t),\quad y(t)=\cos(t)$$

(4) $\frac{d}{dt}f(x(t),y(t),z(t))$, where
$$f(x,y,z)=xy+yz,\quad x(t)=t^2-1,\quad y(t)=2t,\quad z(t)=t^3$$

(5) $\frac{d}{dt}f(x(t),y(t),z(t))$, where
$$f(x,y,z)=x^2yz,\quad x(t)=2t,\quad y(t)=t^3+t,\quad z(t)=t^2-1$$

(6) $\frac{d}{dt}f(x(t),y(t),z(t))$, where
$$f(x,y,z)=\sqrt{x+yz},\quad x(t)=\sin^2(t),\quad y(t)=\cos(t),\quad z(t)=\cos(t)$$

(7) $\frac{d}{dt}f(x(t),y(t),z(t))$, where
$$f(x,y,z)=\ln(1+xyz),\quad x(t)=t^2,\quad y(t)=\frac{1}{t},\quad z(t)=e^t$$

(8) $\frac{\partial}{\partial s}f(x(s,t),y(s,t))$ and $\frac{\partial}{\partial t}f(x(s,t),y(s,t))$, where
$$f(x,y)=x^2+xy,\quad x(s,t)=s+t,\quad y(s,t)=st$$

(9) $\frac{\partial}{\partial s}f(x(s,t),y(s,t))$ and $\frac{\partial}{\partial t}f(x(s,t),y(s,t))$, where
$$f(x,y)=xe^{xy},\quad x(s,t)=s^2t,\quad y(s,t)=s-t$$

(10) $\frac{\partial}{\partial s}f(x(s,t),y(s,t))$ and $\frac{\partial}{\partial t}f(x(s,t),y(s,t))$, where
$$f(x,y)=\sin(\ln(x^2y)),\quad x(s,t)=e^{st},\quad y(s,t)=e^{st^2}$$

Exercise 2. Find the critical points.

(1) $f(x(t),y(t))$, where
$$f(x,y)=xye^y,\quad x(t)=t-2,\quad y(t)=t$$

(2) $f(x(t),y(t))$, where
$$f(x,y)=xy^3-x^2y,\quad x(t)=(t+1)^2,\quad y(t)=t$$

(3) $f(x(t), y(t), z(t))$, where

$$f(x, y, z) = xy + yz, \quad x(t) = -2\sin(t^2), \quad y(t) = t^2, \quad z(t) = t^2\cos(t^2)$$

(4) $f(x(t), y(t), z(t))$, where

$$f(x, y, z) = -2xy+2z+2y^2+4x-15y+13, \quad x(t) = (t-1)^2, \quad y(t) = 2t+1, \quad z(t) = (t-1)^3$$

Exercise 3. Find the distance.

(1) The distance between the point $P = (0, 0)$ and the ellipse

$$x(t) = 2\cos t + \sin t, \quad y(t) = 2\cos t - \sin t$$

(2) The distance between the point $P = (0, 0, 0)$ and the parametric curve

$$x(t) = \ln(t), \quad y(t) = \cos t + \sin t, \quad z(t) = \cos t - \sin t$$

(3) The distance between the point $P = (1, 0, 0)$ and the parametric curve

$$x(t) = t^2, \quad y(t) = \sqrt{3}t, \quad z(t) = -t$$