EXERCISES #15

DIRECTIONAL DERIVATIVES AND THE GRADIENT

Exercise 1. Find the gradient of f.

(1) $f(x,y) = 3x^2y - xy^3$
(2) $f(x,y) = \frac{x}{x+y}$
(3) $f(x,y) = \sqrt{x^2+y^2}$
(4) $f(x,y) = x\ln(x) + y\ln(y)$
(5) $f(x,y) = e^{x\sin(y)}$
(6) $f(x,y,z) = \frac{x}{y+z}$
(7) $f(x,y,z) = x\ln(yz)$
(8) $f(x,y,z) = xyze^{xyz}$

Exercise 2. Find the directional derivative.

(1) $D_{\vec{u}}f(1,1)$, where $f(x,y) = x^2+y^2$ and $\vec{u} = \langle \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}} \rangle$
(2) $D_{\vec{u}}f(3,0)$, where $f(x,y) = x^2e^y$ and $\vec{u} = \langle \frac{3}{5}, -\frac{4}{5} \rangle$.

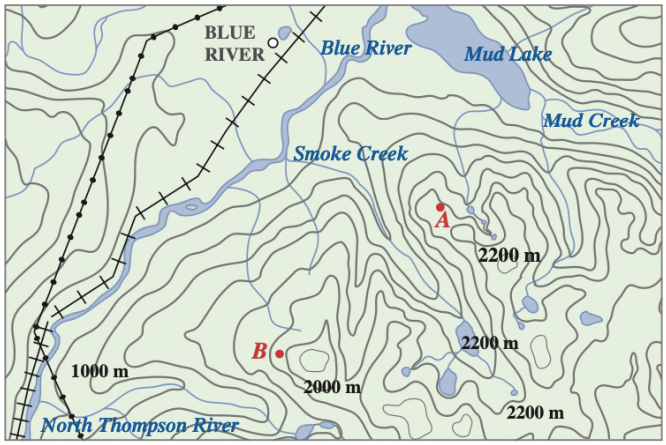
Exercise 3. Find the maximum rate of increase of f at the given point, and the direction in which it occurs.

(1) $f(x,y) = \sin(xy)$ at $(1,0)$.
(2) $f(x,y) = 2xy^2 + xy^3$ at $(1,2)$.
(3) $f(x,y,z) = xyz^2 + x^2y^2$ at $(1,0,-1)$

Exercise 4. Find the tangent plane.

(1) Tangent plane to $xyz = 6$ at $(1,2,3)$
(2) Tangent plane to $x+y+z = e^{xyz}$ at $(0,0,1)$
(3) Tangent plane to $x^4+y^4+z^4 = 3x^2y^2z^2$ at $(1,1,1)$

Exercise 5. Shown is a topographic map of Blue River Pine Provincial Park in British Columbia. Draw curves of steepest descent from point A (descending to Mud Lake) and from point B.


BLUE
RIVER
Blue River
Mud Lake
Mud Creek
Smoke Creek
A
2200 m
B
2200 m
1000 m
2000 m
2200 m
North Thompson River