EXERCISES #19

GLOBAL MAXIMA AND MINIMA III

Exercise 1. List all the nonempty boundary pieces of the domain. Mark every boundary piece that is a bunch of points.

(1) $\{(x, y) \mid 0 \leq x + y \leq 1\}$
(2) $\{(x, y) \mid x^2 + 4y^2 \leq 4,\ x \geq 1\}$
(3) $\{(x, y) \mid x + 2y^2 \leq 0,\ x + y \leq -1\}$
(4) $\{(x, y) \mid 0 \leq x \leq 2,\ 0 \leq y \leq 2\}$
(5) $\{(x, y, z) \mid x^2 + y^2 + z^2 \leq 1,\ x + y \leq 1,\ x \geq \frac{1}{2}\}$
(6) $\{(x, y, z) \mid x^2 + y^2 = z^2,\ x + y \geq 1,\ z \leq 5\}$

Exercise 2. Find the global maximum and minimum values of $f(x, y)$ on the domain D.

(1) $f(x, y) = x^2 + y^2 - 2x$, on the domain D, which is the triangular domain with vertices $(2, 0)$, $(0, 2)$ and $(0, -2)$, including boundaries.
(2) $f(x, y) = x + y + xy$, on the domain D, which is the triangular domain with vertices $(0, 0)$, $(0, 2)$, and $(4, 0)$, including boundaries.
(3) $f(x, y) = x^2 + y^2 + x^2y + 4$, on the domain $D = \{(x, y) \mid -1 \leq x \leq 1,\ -1 \leq y \leq 1\}$.
(4) $f(x, y) = x^2 + xy + y^2 - 6y$, on the domain $D = \{(x, y) \mid -3 \leq x \leq 3,\ 0 \leq y \leq 5\}$.
(5) $f(x, y) = x^2 + 2y^2 - 2x - 4y + 1$, on the domain $D = \{(x, y) \mid 0 \leq x \leq 2, 0 \leq y \leq 3\}$.

Exercise 3. Find the global maximum and minimum values of f on the given domain.

(1) $f(x, y) = x^2y$, on the domain $\{(x, y) \mid x^2 + y^2 = 1,\ y \geq 0\}$.
(2) $f(x, y) = e^{-x^2-y^2}(x^2 + 2y^2)$, on the domain $\{(x, y) \mid x^2 + y^2 = 4,\ x + y \geq 0\}$.
(3) $f(x, y, z) = xyz$, on the domain $\{(x, y, z) \mid x^2 + y^2 + z^2 = 3,\ z \geq 0\}$.

Exercise 4. Find the global maximum and minimum values of f on the given domain.

(1) $f(x, y) = x^3 - 12x + y^3 - 12y$ on the domain

$$D = \{(x, y) \mid (x + 2)^2 + (y + 2)^2 \leq 13,\ x \geq -5\}$$

(2) $f(x, y) = x + y$ on the domain

$$D = \{(x, y) \mid 0 \leq x \leq 1,\ ex^2 \leq y \leq e^x\}$$

(3) $f(x, y, z) = x^4 + y + z^2$ on the domain

$$D = \{(x, y, z) \mid x^2 + y^2 + z^2 \leq \frac{1}{4},\ x \geq 0\}$$

(4) $f(x, y, z) = xz + yz - xy$ on the domain

$$D = \{(x, y, z) \mid z^2 \geq x^2 + y^2,\ z^2 \leq 4\}$$