EXERCISES #20

GLOBAL MINIMA OF COERCIVE FUNCTIONS

Exercise 1. Determine whether the following domain is bounded or not.

(1) $\{(x,y) \mid x^2+y^2 \leq 1\}$
(2) $\{(x,y) \mid x+y=0\}$
(3) $\{(x,y) \mid x^3+y^3 \leq 1\}$
(4) $\{(x,y) \mid x^4+y^2 \leq 1\}$
(5) $\{(x,y) \mid x^2+y^4+x \leq 1,\ y \geq 0\}$
(6) $\{(x,y,z) \mid x^2+y^2+z \leq 1\}$
(7) $\{(x,y,z) \mid x^2+y^4 \leq z^2\}$
(8) $\{(x,y,z) \mid x^2+y^2+z^2 \leq 2x+2y+2z,\ z \geq 0\}$

Exercise 2. Determine whether the following function is a coercive function or not.

(1) $f(x) = x^2$
(2) $f(x) = e^x$
(3) $f(x) = x^2-1$
(4) $f(x,y) = x^4+y^4$
(5) $f(x,y) = e^{x^2+y^2}$
(6) $f(x,y) = e^{x^2-y^2}$
(7) $f(x,y,z) = x^2+y^2+z^2+\sin^2(x)$

Exercise 3. Determine whether f has a global maximum and/or minimum on the domain, and if they exist, find the values.

(1) $f(x,y) = x^2+y^2$, on the domain $\{(x,y) \mid xy \geq 1\}$
(2) $f(x,y) = x^4+y^4$ on the domain $\{(x,y) \mid x^2-y^2 \geq 1\}$
(3) $f(x,y,z) = x^2+y^2+z^2$ on the domain $\{(x,y,z) \mid x-y=1,\ y^2-z^2=1\}$
(4) $f(x,y,z) = x^2+2y^2+3z^2$ on the domain $\{(x,y,z) \mid x+y+z=1,\ x-y+2z=2\}$
(5) $f(x,y,z) = x^2+y^2+z^2$ on the domain $\{(x,y,z) \mid 2x+y+2z=9,\ 5x+5y+7z=29\}$
(6) $f(x,y,z) = x^2+y^2+z^2$ on the domain $\{(x,y,z) \mid z^2=x^2+y^2,\ x+y-z+1=0\}$
(7) $f(x,y,z) = 2x^2+2y^2+z^2+(x-y)^2$ on the domain $\{(x,y,z) \mid xz+yz \geq 4,\ x^2-y^2 \geq 0\}$

Exercise 4. Find all the points on the plane $x+y+z=1$ that are closest to the point $(2,0,-3)$ and compute the distance.

Exercise 5. Find all the points on the plane $x-2y+3z=6$ that are closest to the point $(0,1,1)$ and compute the distance.

Exercise 6. Find all the points on the surface $z=x^2+y^2$ that are closest to the point $(5,5,0)$ and compute the distance.

Exercise 7. Find all the points on the surface $z=x^2+y^2$ that are closest to the point $(1,1,0)$ and compute the distance.

Exercise 8. Find all the points on the surface $xy^2z^3 = 2$ that are closest to the point $(0, 0, 0)$ and compute the distance.