HW #7

CALCULUS III

Question 1. Let

$$f(x,y) = ye^{x\sin(y)}$$

Find an equation of the tangent plane of the surface $z = f(x,y)$ at $(3,0,0)$.

Question 2. Let

$$f(x,y,z) = \ln(1+\sin(x^2+y^2-z^2))$$

Approximate the value of $f(-2.99, 4.01, -4.99)$.

Question 3. Let

$$f(x,y) = xy - x^2, \quad x(s,t) = st + t, \quad y(s,t) = s + \frac{1}{t}$$

By using the Chain Rule, find the partial derivatives

$$\frac{\partial}{\partial s} f(x(s,t), y(s,t)), \quad \frac{\partial}{\partial t} f(x(s,t), y(s,t))$$

Question 4. Recall that the distance between a point P and an object A is the minimum possible distance between P and a random point on A. Using this definition, find the distance between the point $(0,0)$ and the ellipse given by the parametric equation

$$x(t) = 3\cos t + 2\sin t, \quad y(t) = -3\cos t + 2\sin t$$

by going through the steps as follows.

(1) The distance by definition is the global minimum of $f(x(t), y(t))$, where $f(x,y)$ is the distance between $(0,0)$ and (x,y), namely

$$f(x,y) = \sqrt{x^2+y^2}$$

First, find the critical points of $f(x(t), y(t))$.

(2) Then, find the global minimum of $f(x(t), y(t))$ by finding the minimum possible value attained by $f(x(t), y(t))$ at the critical points.