CALCULUS III

Question 1.

(1) Find the maximum rate of increase of
$$f(x, y) = \frac{x}{x^2 + y^2}$$
at the point $P = (-1, 1)$, and the direction in which it occurs.

(2) Find the maximum rate of decrease of $f(x, y)$ at $P = (-1, 1)$, and the direction in which is occurs.

Question 2. Find the unit vectors $\vec{v}$ such that the directional derivative of
$$f(x, y) = x^2 + 2xy^2$$
at the point $P = (1, 1)$ to the direction of $\vec{v}$ has the value 0.

Question 3. The tangent plane of a level surface is orthogonal to the gradient. Using this, find an equation of the tangent plane of the surface
$$xy + yz + zx = 5$$
at the point $P = (1, 2, 1)$.

Question 4. The contour map of a function f is shown. At points P, Q, and R, draw an arrow to indicate the direction of the gradient vector.