Week 2

Deconstruction and geometry

- A new light flashed upon the mind of the first man… who demonstrated the properties of the isoceles triangle… If he is to know anything with *a priori* certainty he must not ascribe to the figure anything save what necessarily follows from what he has himself set into it in accordance with his concept.

Kant, *Critique of Pure Reason*, Preface to Second Edition, p. 19.

- *geometry is a material ontology whose object is determined as the spatiality of the thing belonging to Nature* (Derrida)

(but compare also Husserl, in the *Origin of Geometry*:

geometry (under which title here… we include all disciplines that deal with shapes existing mathematically in pure space-time)

and *Formal and Transcendental Logic*, §30

The great advance of modern mathematics, particularly as developed by Riemann and his successors, consists not in its having merely made clear to itself the possibility of going back in this manner to the form of a deductive system …, starting from geometry and then from other de facto sciences, but rather in its having also gone on *to view such system-forms themselves as mathematical Objects*, to alter them freely, universalize them mathematically, and particularize the universalities

- space and time are the intuitions upon which pure mathematics bases all its cognitions and judgments … Geometry bases itself on the pure intuition of space. …Pure mathematics, as synthetic cognition *a priori*, is possible only because it refers to no other objects than mere objects of the senses, the empirical intuition of which is based on a pure and indeed *a priori* intuition (of space and time), and can be so based because this pure intuition is nothing but the mere form of sensibility…
- … the proposition that not more than three lines can cut each other at right angles in one point… can… by no means be proven from concepts, but rests immediately upon intuition, and indeed on pure *a priori* intuition, because it is apodictically certain. … They [mathematicians who were also philosophers] did not realize that this space in thought itself makes possible physical space, i.e., the extension of matter; that this space is by no means a property of things in themselves, but only a form of our power of sensory representation…

(Kant, *Prolegomena*)

It's hard to answer that question, but I think it's usually taken for granted that something smooth ought to have a tangent space, as well as many other related intuitions; so it's worth pointing out when they fail. Maybe I should have made the analogy with topological (or even homological) manifolds.

> (Peter Scholze, explaining why he said that something he wrote "didn't look very geometric."

Compare Derrida's "invariant structures which have conditioned the advent of geometry"

1. That this pregeometrical world was a world of *things* disposed of according to an anexact space and time.[135]

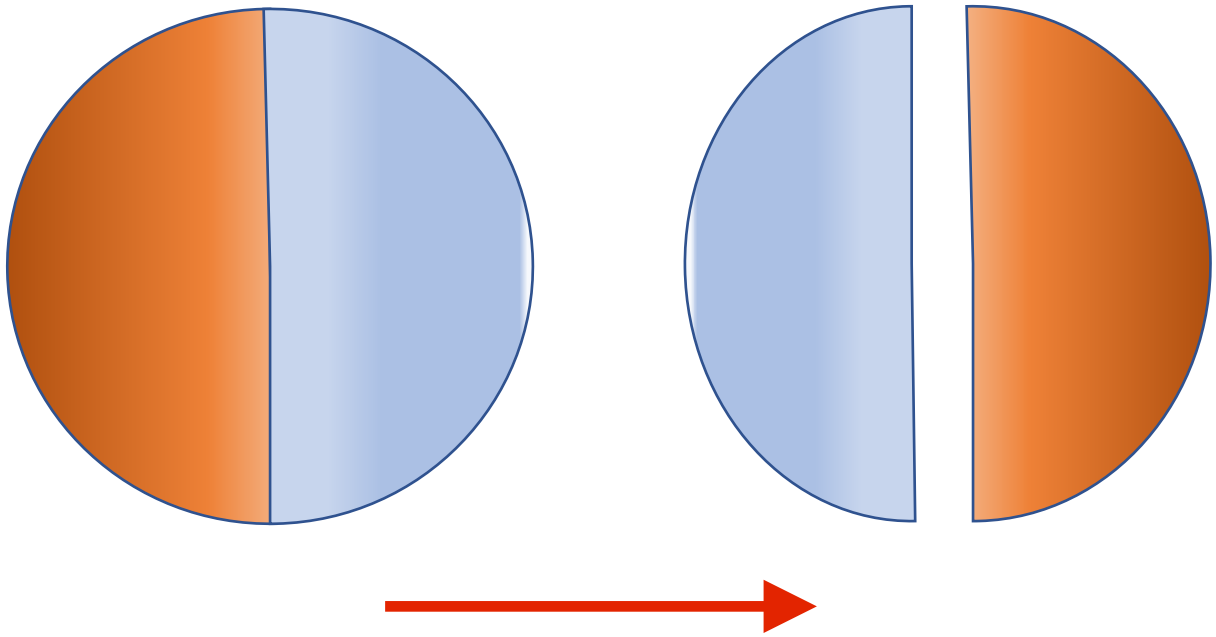
2. That these things must have been "corporeal." Corporeality is a particular determination of thinghood *(Dinglichkeit)* in general; but since culture already had to have left its mark on the world (because language and intersubjectivity must have preceded geometry),[136] corporeality does not exhaustively overlap thinghood: "since the necessarily coexisting human beings are not thinkable as mere bodies and, like even the cultural Objects which belong with them structurally, are not exhausted in corporeal being" (177).

3. That these pure bodies had to have spatial shapes, shapes of motion, and "alterations of deformation" [177].

4. That material qualities (color, weight, hardness, and so forth) must necessarily be "related" to these pregeometrical, spatiotemporal shapes by a supplementary eidetic determination.

- A prime concern for Husserl is the substitution of "logical activities" (in elementary education, for example) for the "reactivation" of the "evident meaning", its "authentic sense of origin" (pp 194-95, see Derrida p. 99 on "alienation technicienne et objectiviste" and p. 100 on "le sens nu de l'évidence fondatrice).

- I want to emphasize that mathematical practice today is oriented, often explicitly, by an effort to pass in the opposite direction — call it "pre-activation," an activity guided by the search for a meaning or a form of intuition that will only be accessible in the future, or that can only be created by transcending intermediate stages of imperfect intuition, and that will retrospectively illuminate the "sedimented" results of the practice of geometry by revealing the true underlying principle of which they were *avatars*.



A discontinuous map: no fixed points

- rather than seeking the conditions for *possible* experience, Deleuze wants to provide an account of the genesis of *real* experience, that is, the experience of this concretely existing individual here and now. Second, to respect the demands of the philosophy of difference, the genetic principle must itself be a differential principle.
  - *Stanford Encyclopedia of Philosophy "Deleuze"*

- *Thereby, the theoretical attitude makes idealization's decisive "passage to the limit" possible, as well as the constitution of the mathematical field in general. Naturally, this passage to the limit is only the going beyond every sensible and factual limit. It concerns the ideal limit of an infinite transgression, not the factual limit of the transgressed finitude.*
- (Derrida, p. 127)

Mazur on discovery vs. invention (platonism vs constructivism.)

- *In the imaginative world, there is an ever unsettling confusion about whether we are bound in nutshells or can count ourselves kings of infinite spaces. On the days when the world of mathematics seems unpermissive, with its gem-hard exigencies, we all become fervid Platonists (mathematical objects are "out there," waiting to be discovered—or not) and mathematics is all discovery. And on days when we see someone who, Viète-like, seemingly by willpower alone, extends the range of our mathematical intuition, the freeness and open permissiveness of mathematical invention dazzle us, and mathematics is all invention.*

- The return to the structures of prescientific experience must continually keep alive the question: How can the a priori Objectivity be constituted starting from those of the life-world?

(Derrida)

> we have been able to demonstrate the incontestable validity of geometry with respect to all objects of the sensible world for the very reason that the latter are mere appearances. …since space as the geometer thinks it is precisely the form of sensory intuition which we find in ourselves a priori and which contains the ground of the possibility of all outer appearances (with respect to their form), these appearances must of necessity and with the greatest precision harmonize with the propositions of the geometer. (Kant, *Prolegomena*)

- *On Husserl's view, it is precisely this "subjective-relative lifeworld", or environment, that provides the "grounding soil" of the more objective world of science (Husserliana, vol. VI, p. 134), in the twofold sense that (i) scientific conceptions owe their (sub-)propositional content and thus their reference to reality to the prescientific notions they are supposed to "naturalize" and that, consequently, (ii) when things get into flux in science, when a crisis occurs, all that is left to appeal to in order to defend new scientific approaches against their rivals is the prescientific lifeworld, as manifested in our according intuitive acceptances ... This view offers an alternative to the "naturalistic" stance taken by many analytic philosophers today.*

(Stanford Encyclopedia, entry on Husserl)

- *Truly, there is not first a subjective geometrical evidence which would then become objective. Geometrical evidence only starts "the moment" there is evidence of an ideal objectivity. The latter is such only "after" having been put into intersubjective circulation. "Geometrical existence is not psychic existence; it does not exist as something personal within the personal sphere of consciousness; it is the existence of what is Objectively there for* 'everyone' (for actual and possible geometers, or those who understand geometry).

(Derrida, ch. 4)

Contrast with to the objectivity of mathematics, which does not become objective as a result of its subjective evidence; It rather develops as the model and prototype of objectivity in general, after having become an intersubjective mode of communication. So how does the expansion of geometry become objective?

Geometry, in effect, is the science of what is absolutely objective— i.e., spatiality—in the objects that the Earth, our common place, can indefinitely furnish as our common ground with other men. But if an objective science of earthly things is possible, an objective science of the Earth itself, the ground and foundation of these objects, is as radically impossible as that of transcendental subjectivity. The transcendental Earth is not an object and can never become one. And the possibility of a geometry strictly complements the impossibility of what could be called a "geo-logy," the objective science of the Earth itself. (Derrida)

- with regard to the objects of mathematics, this sounds like the claim that **objective foundations of mathematics** are impossible. Analytic philosophy took the notion of "objective science of earthly things" seriously by following mathematics in taking the formal logic of propositions as the foundation for such a science, and the axioms of set theory as a proxy for "earthly things."

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It is often traced to Quine's claims that "It is misleading to speak of the empirical content of an individual statement" and that "the unit of empirical significance is the whole of science"

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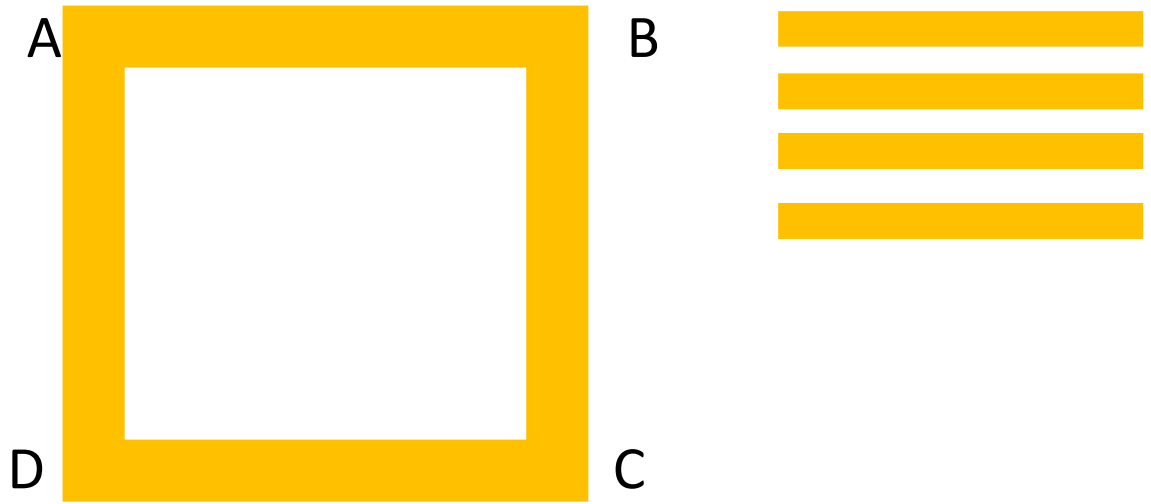
In mathematics the relevant "whole" is something called a *category*, the units are called *objects*, and the relations are called *morphisms*.

- **Yoneda's Lemma**: Any object A in a category is uniquely determined by the sets of morphisms from all other objects B to A.

(Translation: a unit in the system is uniquely determined by its relations to all the other units in the system.)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