ALGEBRAIC NUMBER THEORY W4043

1. Homework, week 6, due October 17

1. Daniel Marcus, *Number Fields*, Exercise 41, pp. 35-36. (This is a long exercise; do as much as you can.)

2. Hindry's book, Exercise 6.15, p. 119. (Use the Vandermonde determinant; please ask the teacher or the TA if this is unfamiliar to you.)

3. The function $D(x_1, x_2, \ldots, x_n)$ in Hindry's exercise 6.15 is called the *discriminant* of the basis $(x_1, \ldots, x_n)$. Compute discriminants of several bases of of the ring of integers in $\mathbb{Q}(\sqrt{d})$, where d is a square-free integer.

4. In the notation of Hindry's exercise, we let p be a prime number, r a positive integer, and let $F(X) = (X^{p^r} - 1)/(X^{p^{r-1}} - 1)$, a polynomial of degree $\phi(p^r)$ where ϕ denotes the Euler function. Let $K = \mathbb{Q}(\zeta)$ be the splitting field of F where ζ is a root of F, and therefore a primitive p^rth root of unity.

(a) Suppose $r = 1$. Show that the discriminant of the basis $\{1, \zeta, \zeta^2, \ldots, \zeta^{p-2}\}$ is equal to $\pm p^{p-2}$.

(b) Determine the sign in (a).

(c) Now for any r, show that the discriminant of the basis $\{1, \zeta, \zeta^2, \ldots, \zeta^{\phi(p^r)-1}\}$ is equal to $\pm p^{p^{r-1}(pr-r-1)}$. (You will find it convenient to use the result of (a).)

5. Use the Minkowski bound to show that the class number (the order of the ideal class group) of $\mathbb{Q}(\sqrt{10})$ is 2. (See exercise 3 on Homework 4.)