

Problem Set VIII for Lie Groups: Fall 2024

October 8, 2024

Problem 1. Give an example of an abelian subgroup of a compact, connected Lie group that is not contained in a torus.

Problem 2. Give a maximal torus of $SO(3)$ and compute the roots and the Weyl group of $SO(3)$. Same question for $SO(4)$.

Problem 3. Show that the adjoint action for S^3 is the double cover $S^3 \rightarrow SO(3)$.

Problem 4. Show that if $A \in SL(2, \mathbb{R})$ is in the image of $\exp: \mathfrak{sl}(2, \mathbb{R}) \rightarrow SL(2, \mathbb{R})$, then either A is diagonalizable or $\text{tr}(A) = 2$. Show that this exponential map is not onto.

Problem 5. Let $g \in G$. Show that the component of the identity of the centralizer $Z(g)$ of g is the union of all the maximal tori that contain g .

Problem 6. Give an example of an element g in a compact connected Lie group G that is contained in only one maximal torus but does not generate that torus.

Problem 7. Classify, up to isogeny all compact Lie groups of dimension 4.

Problem 8. Classify up to isogeny all compact Lie groups of dimension 5.