Representations of finite groups, spring 2016
Homework 10, due Wednesday April 13 before class.

1. (10 points) The alternating group A_3 (isomorphic to the cyclic group $\mathbb{Z}/3$) is a subgroup of S_3. For each irreducible representation V_i, $i = 0, 1, 2$ of A_3 compute the character of the induced representation $\mathrm{Ind}(V_i)$ of S_3 and use it to decompose $\mathrm{Ind}(V_i)$ into irreducibles. Write down the matrix of the induction functor. What is the matrix of the adjoint restriction functor?

2. (10 points) Let $F : \mathcal{A} \longrightarrow \mathcal{B}$ and $G : \mathcal{B} \longrightarrow \mathcal{C}$ be functors between categories $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$. Give a definition of the composition $G \circ F : \mathcal{A} \longrightarrow \mathcal{C}$ and check that it is a functor.

3. (20 points) (a) Consider the inclusion functor $G : Ab \longrightarrow Gr$ from the category of abelian groups to the category of groups. Show that G has a left adjoint functor F and describe F explicitly.

(b) Consider the forgetful functor $G : Top \longrightarrow Set$ from the category of topological spaces to the category of sets. Show that G has both left and right adjoint functors and find these two functors.

4. (20 points) For each irreducible representation V_i of the symmetric group S_4 compute characters of symmetric and exterior squares $S^2(V_i)$ and $\Lambda^2(V_i)$ and determine how they decompose into irreducibles.

5. (10 points) Show that $\Lambda^2(V^*) \cong (\Lambda^2 V)^*$, for any representation V of a finite group (hint: compare the characters of these representations).