Representations of finite groups, spring 2016

Homework 5, due Monday February 29 before class.

1. (a) Describe all (complex) irreducible representations of the Klein group $G = C_2 \times C_2$.

(b) Can you find the four non-trivial idempotents in the group algebra $\mathbb{C}[G]$ that add up to 1? What is the relation of these idempotents to the irreducible representations of G?

For computations with idempotents and characters, it's often best to write groups, even abelian, in multiplicative form. For instance, G above has generators g, h and defining relations $g^2 = h^2 = 1$, $gh = hg$.

2. Prove that a finite group G is abelian if every complex irreducible representation of G is one-dimensional. (Recall that in class we proved that for finite abelian G any complex irreducible representation of G is one-dimensional.)

3. Let V be the 2-dimensional complex representation of $\mathbb{Z}/3$ where the generator g of $\mathbb{Z}/3$ acts on the basis vectors by

$$g(v_1) = v_2, \qquad g(v_2) = -v_1 - v_2.$$

Express V explicitly as a direct sum of irreducible representations. Compute χ_V.

4. Prove that the character χ_V is multiplicative,

$$\chi_V(gh) = \chi_V(g)\chi_V(h), \qquad \forall g, h \in G,$$

if and only if V is one-dimensional.

5. Verify orthogonality relations for characters of irreducible representations of cyclic groups $\mathbb{Z}/2$ and $\mathbb{Z}/3$.

6. Check orthogonality relations for characters of irreps of S_3 using the character table derived in class.

7. Give an example of a representation V of the infinite cyclic group $\mathbb{Z}$ such that $\chi_V(g) = \frac{1}{2}$, where g is a generator of $\mathbb{Z}$.