Representations of finite groups, spring 2016
Homework 8, due Wednesday March 30 before class.

Reading: Sections 7.1, 7.2 of Steinberg (pages 83-90). Topics: permutation representations, tensor products, dual representation.

1-2. Do exercises 7.2 and 7.5 in Steinberg, pages 94-95.

3. For which of the following group actions is the augmentation representation irreducible (use methods developed in Steinberg Section 7.2)? Your answer might depend on the value of the parameter n; then explain the dependence on n.

(a) Permutation action of the symmetric group S_n on an n-element set.

(b) Restriction of this permutation action to the alternating subgroup A_n, $n > 1$.

4. Consider the augmentation representation V for the action of a finite group G on itself by left multiplication. Prove that V is irreducible if and only if G has order 2.

5. We classified irreducible representations of S_4 in class and denoted them V_0, V_1, V_2, V_3, V_4. Recall the character table of S_4 and write it down. Determine multiplicities of irreducible representations in various tensor products $V_i \otimes V_j$, for $i, j = 0, 1, 2, 3, 4$. Use characters and properties of tensor products.

6. The quaternion group Q_8 has a unique irreducible 2-dimensional representation V. What are the multiplicities of irreducible representations in $V \otimes V$? In $V \otimes V \otimes V$? You will need to recall the character table of Q_8.

7. (a) Show that the trivial representation of any group is self-dual.

(b) For each irreducible representation of $\mathbb{Z}/4$ determine its dual. How many irreducible representations of $\mathbb{Z}/4$ are self-dual?

(c) Among the groups listed below select those with every irreducible presentation being self-dual:

$$\mathbb{Z}/2, \qquad S_5, \qquad Q_8, \qquad \mathbb{Z}_6, \qquad \mathbb{Z}/2 \times \mathbb{Z}/2 \times \mathbb{Z}/2.$$