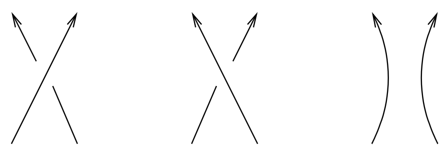
Introduction to knot theory, Spring 2012

Homework 10, due Monday, April 9

The Alexander polynomial in $q = \sqrt{t}$ is determined by the normalization $\Delta(\text{unknot}) = 1$ and by the skein relation

$$\Delta(L_+) - \Delta(L_-) = (q^{-1} - q)\Delta(L_0)$$

where L_+, L_-, L_0 are any three links which differ only inside a small ball, where L_+ has a positive crossing, L_- a negative crossing, and L_0 a smoothing, with the strands oriented up.



L_+ L_- L_0

1. (10 points) Explain why these properties are enough to determine the Alexander polynomial of any link.

2. (10 points) Make a substitution $z = q^{-1} - q$. Explain why the Alexander polynomial of any link is a function of z only. Writing $\Delta(L)$ as a function of z gives a polynomial called the Conway polynomial $\nabla(L)$.

3. (10 points) Write down the skein relation for the Conway polynomial. Use it to show that the Conway polynomial of any split link is 0.

4. (30 points) Using the skein relation, compute the Conway polynomial of the Hopf link, the trefoil, figure-eight knot, and the Whitehead link.