Introduction to knot theory, Spring 2012

Homework 11, due Wednesday, April 18

1. (20 points) Using relation between integral homology group, homology groups with coefficients in a field F, and integral cohomology groups derived in class, determine homology groups with coefficients in $\mathbb{Z}/2, \mathbb{Z}/3, \mathbb{Q}$ and integral cohomology of the following surfaces:

annulus, Möbius band, two-sphere, two-torus, projective plane, Klein bottle.

2. (20 points) A closed connected oriented surface M of genus g has a cell decomposition with one zero-cell, $2g$ one-cells, and one two-cell. Using this decomposition write down the complex which computes homology groups of M with coefficients in any field F and find these groups. Via duality between homology groups and cohomology groups, determine cohomology groups of M with coefficients in F.

3. (10 points) Find integral cohomology groups of the complement $\mathbb{S}^3 \setminus L$ of an n-component link L in $\mathbb{S}^3$.

3.(20 points) Topological space X has the following integral homology groups:

$$\mathrm{H}_0(X, \mathbb{Z}) = \mathbb{Z}, \quad \mathrm{H}_1(X, \mathbb{Z}) = \mathbb{Z} \oplus \mathbb{Z}/4, \quad \mathrm{H}_2(X, \mathbb{Z}) = \mathbb{Z}/6.$$

Determine homology groups of X with coefficients in $\mathbb{Q}$, in $\mathbb{Z}/2$, and in $\mathbb{Z}/3$. What are integral cohomology groups of X? Determine cohomology groups of X with coefficients in $\mathbb{Q}$, in $\mathbb{Z}/2$, and in $\mathbb{Z}/3$.

4. (10 points) Simplify the following tensor products of abelian groups: $\mathbb{Z} \otimes \mathbb{Z}/4$, $\mathbb{Z}/4 \otimes \mathbb{Z}/6$, $\mathbb{Z}/6 \otimes \mathbb{Z}/18$, $(\mathbb{Z} \oplus \mathbb{Z}/3) \otimes \mathbb{Z}/3$, $(\mathbb{Z} \oplus \mathbb{Z}/9) \otimes (\mathbb{Z} \oplus \mathbb{Z}/4)$.