Introduction to knot theory, Spring 2012

Read section 4 of Knotes.

Homework 3, due Monday, February 13

1. (20 points) Exercise 4.1.6 in Knotes (page 23).

2. (20 points) We computed in class the Jones polynomial of the left-handed Hopf link and the left-handed trefoil (closure of braid σ_1^{-3}). Compute the Jones polynomial of the right-handed Hopf link (closure of braid σ_1^2) and the right-handed trefoil (closure of σ_1^3) via the skein relation and via the Kauffman bracket (the answers should agree). Compare your answers with the conjugates of Jones polynomials of the right-handed Hopf link and trefoil (the latter were computed in class; the conjugate of a polynomial is obtained by substituting q^{-1} for q in it).

3. (10 points) Compute the Jones polynomial $J(4_1)$ of the figure-eight knot via the skein relation. Is it self-conjugate?

4. (20 points) Prove that $J(K_1 \# K_2) = J(K_1)J(K_2)$ and

$$J(K_1 \sqcup K_2) = J(K_1)J(K_2)(-q - q^{-1}).$$

Here $K_1 \sqcup K_2$ is the disjoint union of links K_1 and K_2.

5. (10 points) Using the Jones polynomial, show that the knots $3_1 \# 3_1$ and $3_1 \# 3_1^!$ are distinct.

Extra credit: Knotes exercises 4.4.12-13.