Introduction to knot theory, Spring 2012

Read "Catalan numbers", by Tom Davis.

Homework 4, due Monday, February 20

1. (20 points) Show that the value of the Jones polynomial $J(K)$ at $A = 1$ depends only on the number of components of K. Determine this value as the function of the number of components. What can you say about the value of $J(K)$ at $A = -1$?

2. (20 points) Draw all crossingless matchings with 8 endpoints. How many of them are there? Draw the mountain range (Dyck path) corresponding to each crossingless matching.

3. (20 points) Show that the n-the Catalan number is the number of root binary trees with n internal nodes (see Section 1.6 of "Catalan numbers" for details and examples).

4. (20 points) (a) Compute the multiplication table for TL_3 in the monomial basis $\{1, u_1, u_2, u_1u_2, u_2u_1\}$.
(b) Show that the Temperley-Lieb algebra TL_n is commutative if and only if $n \leq 2$.

5. (20 points) Write down all elements in the monomial basis of TL_4. There must be a natural bijection between these elements and crossingless matchings from problem 2.

Extra credit: Choose a problem from "Catalan addendum" by R. Stanley that was not discussed in class (most of them weren't). Either solve it yourself or look up the proof. Write up a solution of the problem. If "Catalan addendum" appears too complex, you can instead solve problem in section 7.2 of "Catalan numbers" - show that there are exactly c_n ways to tile the staircase with n steps by n rectangles.