Introduction to knot theory, Spring 2012

Homework 8, due Monday, March 26

1. (20 points) Read and think through the proof of exactness of the complex of homology groups associated with a short exact sequence of complexes (Theorem 2.16 in Hatcher). Choose two out of six cases from the proof and write them up in your own words.

2. (10 points) Give an example of short exact sequence of complexes $0 \longrightarrow A \longrightarrow B \longrightarrow C \longrightarrow 0$ such that the boundary map $\partial : H_n(C) \longrightarrow H_{n-1}(A)$ is nontrivial for some n.

3. (10 points) How does Mayer-Vietoris sequence simplify when $A \cap B = \emptyset$?

4. (10 points) Draw trefoil 3_1, its meridian and longitude. Do the same for the figure-eight knot 4_1.

5. (10 points) Explain why the longitude of a knot can be obtained by slightly pushing the knot into the interior of its Seifert surface.

6. (20 points) Let R be a commutative ring and M an R-module which admits a finite presentation.

(a) Show that the first elementary ideal $\mathcal{E}_1(M)$ does not depend on the choice of presentation (this is true for all elementary ideals, not just the first one).

(b) Show that $\mathcal{E}_{r-1}(M) \subset \mathcal{E}_r(M)$.

(c) Compute elementary ideals of the following $\mathbb{Z}$-modules: $\mathbb{Z}/10 \oplus \mathbb{Z}/4$, $\mathbb{Z} \oplus \mathbb{Z}/3$, $\mathbb{Z} \oplus \mathbb{Z}$.