Introduction to knot theory. Practice quiz.

Mark the squares that are followed by correct statements.

□ Any knot in $\mathbb{S}^3$ bounds an unorientable surface.

True. The boundary of the Möbius band, with its usual embedding in $\mathbb{R}^3$, is the unknot. Take the Seifert surface for a knot (which is oriented), or the surface obtained from the checkerboard coloring of a knot's projection (which may or may not be oriented), and form the band-connected sum with the above Möbius band, producing an unorientable surface bounding the knot.

□ Any two-component link with the linking number 0 is a split link.

False. For instance, the Whitehead link (Knot notes, page 3) has linking number 0, but is not split. Exercise 3.3.8 (page 16) shows that it's not the unlink, similar computation shows that it's not a split link.

□ The only knot which has the trivial Alexander polynomial is the unknot.

False. The Whitehead double of any knot has Alexander polynomial 1.

□ There exists a knot with the Alexander polynomial $t - 2 + t^{-1}$.

False. The Alexander polynomial of any knot satisfies $\Delta_K(1) = 1$.

□ The Alexander polynomial of the connected sum of two knots is the product of their Alexander polynomials.

True.

☐ If a knot has Alexander polynomial $t^2 - 2t + 3 - 2t^{-1} + t^{-2}$, then its genus is at least 2.

True. The width of the Alexander polynomial is a lower bound on twice the genus of the knot.

☐ If $K = K\#K$, then K is the trivial knot.

True. Follows from the uniqueness of the prime decomposition for knots.

☐ A knot can have the Seifert matrix of size 4×4 with all entries being 0.

False. The Alexander polynomial of a knot satisfies $1 = \Delta_K(1) = \det(A - A^T)$, where A is the Seifert matrix.

☐ Any oriented link in $\mathbb{S}^3$ bounds a connected orientable surface.

True.

☐ For any Seifert surface F, the first homology group $H_1(\mathbb{S}^3 \setminus F, \mathbb{Z})$ is a free abelian group.

True.

☐ Any link diagram D can be turned into the diagram of an unlink by reversing several crossings, if necessary.

True.

☐ The 0-th homology group $H_0(F, \mathbb{Z})$ of any surface F is isomorphic to $\mathbb{Z}$.

False. If F is connected, then indeed $H_0(F, \mathbb{Z}) \cong \mathbb{Z}$. In general, $H_0(F, \mathbb{Z})$ is the direct sum of $\mathbb{Z}$'s, one for each connected component of F.