## Lie groups and representations, Fall 2009

## Homework #2, due Wednesday, Sept 30.

1. Prove that  $\mathbb{C} \otimes_{\mathbb{R}} \mathbb{C} \cong \mathbb{C} \times \mathbb{C}$ .

2. (a)Classify irreducible representations of  $\mathbb{Z}/n$  over reals. For each representation, determine whether it's real, complex or quaternionic. (b) Do the same for  $\mathbb{Z}/3 \times \mathbb{Z}/3$ .

3. Prove that any complex irreducible representation of a finite group G has dimension one or two if G contains an abelian subgroup of index 2. Conclude that any irrep of a dihedral or a binary dihedral group is at most two-dimensional.

4. Determine the Frobenius matrix of induction-restriction multiplicities between irreducibles for the inclusion of symmetric groups  $S_3 \subset S_4$ .