Topology

Homework #1. Due Monday, September 18, before the lecture.

Read the following parts of Chapter 2: §12, §13, §15, §17 and the first half of §16.

Solve the following problems:

Exercises 2, 7 and 8a on page 83 of the textbook (skip $\mathcal{T}_2$ in exercise 7).

Exercises 2, 3, 6 on page 92.

1. Which of the following are topologies on $\mathbb{R}$?

- Any set is open.
- Only the empty set is open.
- U is open if either $U \cap \mathbb{N} = \emptyset$ or U contains all but finitely many natural numbers ($\mathbb{N}$ denotes the set of natural numbers).
- U is open if either $U = \emptyset$ or U is an infinite subset of $\mathbb{R}$.

2. (a) Let X and Y be topological spaces such that X and Y are disjoint sets. We declare that $U \subset (X \sqcup Y)$ is open if $U \cap X$ is open in X and $U \cap Y$ is open in Y. Check that we get a topology on the disjoint union of X and Y.

(b) Prove that for X and Y as above, $X \sqcup Y$ is Hausdorff if and only if both X and Y are Hausdorff.

3. Suppose that X is Hausdorff and Y is a subset of X with the induced topology. Show that Y is Hausdorff.

4. Find the closure and interior of each of the following subsets of $\mathbb{R}^2$:

(a) $A = \{(x, y) | y = 0 \text{ and } x < 0\}$,
(b) $B = \mathbb{R}^2$,
(c) $C = \{(x, y) | x > 0 \text{ and } y > 0\}$,
(d) $D = \{(x, y) | x \in \mathbb{Q}\}$.