Topology

Homework #3. Due Monday, October 2, before the lecture.

Read §14, §26, the first half of §27, and do the following problems.

Exercises 2a and 3 on page 171.

In exercises 1, 2 and 3 below do not justify your answers.

1. Which of the following subsets of $\mathbb{R}$ are (a) closed, (b) compact?

- $[0, 1]$
- $(-1, 1]$
- $\{0\} \cup \left\{\frac{1}{n}\right\}_{n \geq 1}$
- $\mathbb{Z}$
- The empty set $\emptyset$
- $[-2, -1) \cup (-1, 0]$
- the set of all irrational numbers between 1 and 2
- the set of all integers between -2006 and 2006

Determine the interior and the limit point set for each set above.

2. Which of the following subsets of $\mathbb{R}^2$ are (a) closed, (b) compact?

- $\{(x, y) | -1 \leq x \leq 1, 0 < y \leq 2\}$.
- Set of points (x, y) with at least one coordinate an integer.
- The set of points at distance ≤ 1 from the origin.
- $\{(x, y) | x \geq 0, y \geq 0, x + y \leq 1\}$.
- $\{(3, 3), (1, -4)\}$.
- $\{(x, y) | x + y \leq 1\}$.
- $\{(x, y) | 1 \leq xy \leq 3\}$.
- $\{(x, y) | -2 \leq x \leq 1, y = 3\}$.

- The union of concentric circles of radii $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$, all centered at 0 and the point $(0,0)$. We can write this set as

$$\{(x,y)|x^2+y^2=\frac{1}{n^2}, n\in\mathbb{N} \text{ or } x=y=0.\}$$

3. Among the statements below, select those that are correct.

a. Any T_1-space is Hausdorff.

b. The complement in $\mathbb{R}$ of the Cantor set is closed.

c. The discrete topology is metrizable.

d. The empty set is both open and closed in any topological space X.

e. Any continuous map between discrete spaces is a homeomorphism.

f. The composition of homeomorphisms is a homeomorphism.

g. If each space $X_i, i\in\mathbb{N}$, has the discrete topology, then the product topology on $X_1\times X_2\times\ldots$ is discrete.

h. Any two open intervals in $\mathbb{R}$ are homeomorphic.

i. Any subspace of a compact space is compact.

j. If $f: X\longrightarrow Y$ is a continuous map and $Z\subset X$ has the induced topology, then the restriction of f to Z is continuous.