Topology

Homework #6. Due Monday, October 30, before the lecture.

1. (a) Compute 5-adic norms of 225, $\frac{-6}{15}$, $\frac{3}{467}$, -4.

(b) Find 5-adic expansions of $\frac{-1}{3}$ and $\frac{5}{4}$.

2. Which of the following sequences $\{x_n\}$ are Cauchy in $\mathbb{Q}$ with respect to the 7-adic metric?

$$x_n = 7^n, \quad x_n = \frac{1}{7^n}, \quad x_n = n, \quad x_n = 1 - n \cdot 14^n, \quad x_n = -2^n.$$

3. Prove that if G and H are topological groups, then their direct product is a topological group (first solve exercise 10 on page 112).

4. Consider the set $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ of nonzero complex numbers and equip it with the topology induced from that of $\mathbb{C}$. Show that $\mathbb{C}^*$ is a topological group (the group operation is the multiplication of complex numbers).