Topology

Homework #7. Due Wednesday, November 8, before the lecture.

Exercise 4 on page 92.

1. (a) Show that a continuous map $f : X \longrightarrow Y$ is open if, for some basis B of X, the set $f(U)$ is open in Y for any $U \in B$.

(b) Give an example of a continuous surjective map from $[0, 1]$ to itself which is not open.

2. Prove that a continuous open bijective map $f : X \longrightarrow Y$ is a homeomorphism.

3. Which of the following maps are open? (briefly justify your answer)

- Inclusion of the open unit disk into $\mathbb{R}^2$.
- Inclusion of the closed unit square into $\mathbb{R}^2$.
- The map that takes $\mathbb{R}$ to the point 5 of $\mathbb{R}$.
- The map from $\mathbb{R}^2$ to $\mathbb{R}$ which takes (x, y) to $x + y$.

4. Consider the equivalence relation $\sim$ on $[0, 1]$ which partitions $[0, 1]$ into three equivalence classes: $\{0\}, (0, 1)$, and $\{1\}$. Explicitly describe the quotient topology on $[0, 1]^*$. Is this topology T_1?

5. Which of the following spaces are contractible? (briefly justify your answer)

- $\mathbb{R}^n$
- The space of irrational numbers with the topology induced from $\mathbb{R}$
- $\{(x, y) \in \mathbb{R}^2 | 0 < x \leq 1\}$
- $\{(x, y) \in \mathbb{R}^2 | 2 < x - y < 3\}$
- $\mathbb{R} \setminus \{0\}$

6. Prove that the product $X \times Y$ of two contractible spaces is contractible.