Topology, fall 2022.

NAME:

Quiz 1

I (11 points). Mark the boxes that are followed by correct statements.

1) ☐ Collection of sets $\{\emptyset, \{b\}, \{a, b, c\}\}$ is a topology on the set $\{a, b, c\}$.

2) ☐ If $\mathcal{B}_1, \mathcal{B}_2$ are both bases for a topology $\mathcal{T}$ on X then their union $\mathcal{B}_1 \cup \mathcal{B}_2$ is also a basis for the topology $\mathcal{T}$.

3) ☐ If X, Y have indisrete topologies, the product topology on $X \times Y$ is indiscrete.

4) ☐ Indiscrete topology is finer than any other topology on a set X.

5) ☐ The ordered square I_o^2 is Hausdorff.

6) ☐ A finite topological space is Hausdorff if and only if it is discrete (carries discrete topology).

7) ☐ Set $\mathbb{N}$ of natural numbers with the finite complement topology is Hausdorff.

8) ☐ If X is a metric space, any subset $Y \subset X$ inherits a metric from X.

9) ☐ The interval $[0, \pi]$ with the distance function $d(a, b) = \sin|a - b|$ is a metric space.

10) ☐ Topological space $X = \{a, b, c\}$ with the topology $\{\emptyset, \{c\}, \{b, c\}, X\}$ is connected.

11) ☐ If both $X \cup Y$ and Y are connected then X is connected.

II (5 points). Mark the square in the *first* column, respectively **second** column, if the corresponding subset of $\mathbb{R}^2$ with the standard topology is *open*, respectively **closed**.

☐ ☐ $\{(x,y)|x \geq 0 \text{ or } y \geq 0\}$
☐ ☐ $\{(x,y)|x < 0 \text{ and } y \geq 0\}$
☐ ☐ $\{(x,y)|x = 1 \text{ and } y \leq 2\}$
☐ ☐ $\{(x,y)|xy = 1\}$
☐ ☐ $\{(x,y)|x^2 + y^2 \geq 1\}$

III (3 points) Mark the boxes that are followed by correct statements.

a) ☐ Sequence $x_n = (\frac{1}{n}, \frac{1}{2n}, \frac{1}{3n}, \dots)$ converges to $\underline{0} = (0,0,0,\dots)$ in the uniform topology on $\mathbb{R}^{\mathbb{N}}$.

b) ☐ The identity map $\mathbb{R} \longrightarrow \mathbb{R}_\ell$ from $\mathbb{R}$ with the standard topology to $\mathbb{R}$ with the lower limit topology is continuous.

c) ☐ The identity map $\mathbb{R}^{\mathbb{N}} \longrightarrow \mathbb{R}^{\mathbb{N}}$ from $\mathbb{R}^{\mathbb{N}}$ with the box topology to $\mathbb{R}^{\mathbb{N}}$ with the product topology is continuous. Here each $\mathbb{R}$ in this infinite product carries the standard topology.