Topology, fall 2022.

Quiz 1 Solutions

I (11 points). Mark the boxes that are followed by correct statements.

1) ■ Collection of sets $\{\emptyset, \{b\}, \{a, b, c\}\}$ is a topology on the set $\{a, b, c\}$.
True. This set is closed under finite intersections and arbitrary unions, contains the empty set and the entire space.

2) ■ If $\mathcal{B}_1, \mathcal{B}_2$ are both bases for a topology $\mathcal{T}$ on X then their union $\mathcal{B}_1 \cup \mathcal{B}_2$ is also a basis for the topology $\mathcal{T}$.
True. This is a good exercise. Check basis axioms for $\mathcal{B}_1 \cup \mathcal{B}_2$.

3) ■ If X, Y have indisrete topologies, the product topology on $X \times Y$ is indiscrete.
True. Use the definion of the product topology to check this.

4) □ Indiscrete topology is finer than any other topology on a set X.
False. It's the opposite, in fact. The indiscrete topology is coarser than any topology on X.

5) ■ The ordered square I_o^2 is Hausdorff.
True. We proved that any order topology is Hausdorff.

6) ■ A finite topological space is Hausdorff if and only if it is discrete (carries discrete topology).
True. We discussed this briefly in class. A topology is T_1 if points are closed. In a finite topological space points are closed iff X is discrete (since then any subset $Y \subset X$ is closed).

7) □ Set $\mathbb{N}$ of natural numbers with the finite complement topology is Hausdorff.
False. The finite complement topology on an infinite set is not

Hausdorff.

8) ■ If X is a metric space, any subset $Y \subset X$ inherits a metric from X.
True.

9) □ The interval $[0, \pi]$ with the distance function $d(a, b) = \sin|a - b|$ is a metric space.
False. What is the distance $d(0, \pi)$ in this topology?
Suppose you restrict to the open interval $(0, \pi)$. Does that distance function define a metric?

10) ■ Topological space $X = \{a, b, c\}$ with the topology $\{\emptyset, \{c\}, \{b, c\}, X\}$ is connected.
True. There exists no separation of X.

11) □ If both $X \cup Y$ and Y are connected then X is connected.
False. For a counterexample, take a connected Y and $X \subset Y$ not connected. Say $Y = \mathbb{R}$ and $X = \{0, 1\}$.

II (5 points). Mark the square in the *first* column, respectively **second** column, if the corresponding subset of $\mathbb{R}^2$ with the standard topology is *open*, respectively **closed**.

□ ■ $\{(x, y)|x \geq 0 \text{ or } y \geq 0\}$
□ □ $\{(x, y)|x < 0 \text{ and } y \geq 0\}$
□ ■ $\{(x, y)|x = 1 \text{ and } y \leq 2\}$
□ ■ $\{(x, y)|xy = 1\}$
□ ■ $\{(x, y)|x^2 + y^2 \geq 1\}$

III (3 points) Mark the boxes that are followed by correct statements.
a) ■ Sequence $x_n = (\frac{1}{n}, \frac{1}{2n}, \frac{1}{3n}, \dots)$ converges to $\underline{0} = (0, 0, 0, \dots)$

in the uniform topology on $\mathbb{R}^{\mathbb{N}}$.

True. Take a basis neighbourhood $B(\underline{0}, \epsilon)$, $\epsilon > 0$ and check that all x_n starting with some n are in that neighbourhood.

b) ☐ The identity map $\mathbb{R} \longrightarrow \mathbb{R}_\ell$ from $\mathbb{R}$ with the standard topology to $\mathbb{R}$ with the lower limit topology is continuous.

False. $\mathbb{R}_\ell$ is strictly finer than the standard topology and has more open sets, so that not every inverse image of an open set is open.

c) ☐ The identity map $\mathbb{R}^{\mathbb{N}} \longrightarrow \mathbb{R}^{\mathbb{N}}$ from $\mathbb{R}^{\mathbb{N}}$ with the box topology to $\mathbb{R}^{\mathbb{N}}$ with the product topology is continuous. Here each $\mathbb{R}$ in this infinite product carries the standard topology.

True. The box topology is finer than the product topology.