

Topology, fall 2022

Homework 3, due Thursday September 29.

Read Munkres sections §19 – 20 (we'll talk about the box topology on Tuesday next week).

I-III. Solve problems 6, 8a, 12 on pages 111-112 (exercise 11 introduces terminology needed for problem 12).

VI. Prove that the Cantor set C is homeomorphic to the disjoint union of two copies of itself and to the direct product of two copies of itself:

$$C \cong C \sqcup C, \quad C \cong C \times C.$$

Show that discrete infinite countable topological space $\mathbb{N} = \{1, 2, \dots\}$ satisfies the same properties:

$$\mathbb{N} \cong \mathbb{N} \sqcup \mathbb{N}, \quad \mathbb{N} \cong \mathbb{N} \times \mathbb{N}.$$

(Of course, C is a more interesting example than \mathbb{N} . For instance, we'll see soon that C is compact.)

V-VI. Solve exercises 1a, 2. For exercise 2, first try to figure out the topology on $\mathbb{R} \times \mathbb{R}$ in the dictionary order and find a simpler way to think of it as a topological space.

VII. Exercise 4ab on page 127 (either read ahead about the uniform and box topologies or wait till Tuesday when we'll discuss them.)