Topology, fall 2022

Homework 9, due Tuesday, Nov 22. (60 points)

Read Munkres Sections $\S 22, 55, 56$.

I. Exercise 8 on page 348.

II. Drew two different path-connected coverings of the figure eight space $\mathbb{S}^1 \vee \mathbb{S}^1$ with five sheets each (degree five coverings). See the printout from Hatcher's book for other examples of coverings of $\mathbb{S}^1 \vee \mathbb{S}^1$.

III. Suppose X is a topological space which is a finite tree graph. Prove that X is contractible. Does your proof generalize to arbitrary (infinite) trees?

IV. Exercise 1 on page 353.

V. Exercise 1 on page 356.

VI. Consider the subspace $X \subset \mathbb{R}^2$ of points (x, y) such either x or y is an integer (this subspace is the square grid in the plane). Which of the following subspaces of X are retracts of X?
(a) Perimeter of the unit square (four edges in X that constitute a square),
(b) $A = [0, 2] \times \{0\} \subset \mathbb{R}^2$.
(c) $A = (0, 3) \times \{1\} \subset \mathbb{R}^2$.
(d) $A = \{(0, 0), (0, 1), (1, 1)\}$ (union of three points).
(e) A is the union of the x-axis and the y-axis.

Additional problems to think about: Exercise 2 on page 353 (hint: lift h to a continuous map from $\mathbb{S}^1$ to $\mathbb{R}$), exercise 4 on page 353. Exercise 2 on page 356.

1. For each of the following spaces X, find a continuous map $X \longrightarrow X$ without fixed points:
(a) $X = \mathbb{S}^1$ (a circle),
(b) $X = \mathbb{S}^1 \times \mathbb{S}^1$ (a 2-torus),
(c) $X = \mathbb{R}^3$,
(d) X is the Cantor set,
(e) $X = [0, 1] \cup \{2\}$ (union of a closed interval and a point).