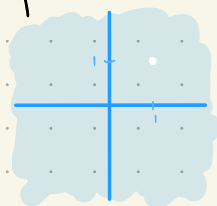
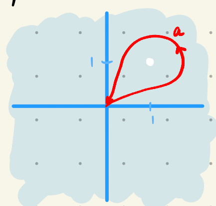
14. The fundamental group of a space.

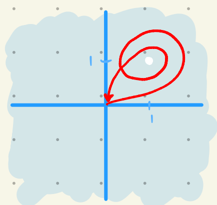
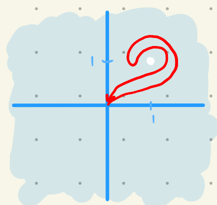
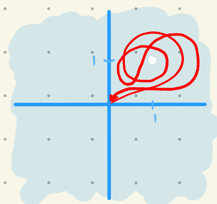
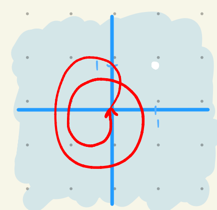
1. Consider again the space $X =$ 

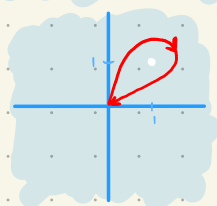
   a) Identify the identity element $1 \in \pi(X)$ (i.e. a path such that for any other path x, $x \circ 1 = 1 \circ x = x$)

   Consider the path 

   , and call it a. Denote by a^n the concatenation of a with itself n times, and write $a^0 = 1$.

   b) Express the following paths as powers of a:

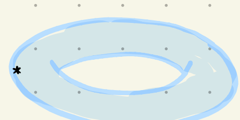
      

   c) What should the path  be in terms of a?

   d) Given any path $x \in X$, what should x^{-1} be?

   e) Convince yourselves that every element in $\pi_1(X)$ is in fact a power of a.

   f) Define an isomorphism $\pi_1(X) \to \mathbb{Z}$ (You don't have to prove that it is an isomorphism).

2. Let X be the torus 

   . Find two distinct paths a, b based at $*$ such that $ab = ba$.

3. (Challenge) Find a space X such that $\pi_1(X) \cong C_2$.