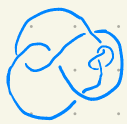
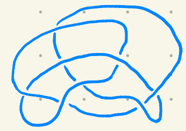
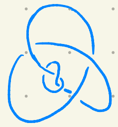
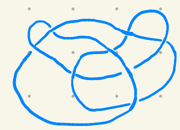
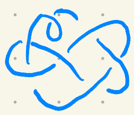
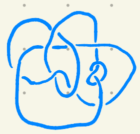
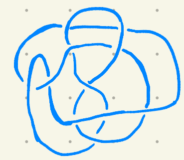
3. Multiplying Knots, the table of prime Knots.

1. Classify the following Knots into prime or composite. For the composite ones K, identify K_1 and K_2 such that $K = K_1 \# K_2$. (Here $K_1, K_2 \neq$ unknot). You may use the table of Knots in the next page.

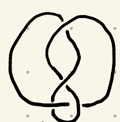
a1)  a2)  a3) 

b1)  b2)  b3) 

c1)  c2)  c3) 

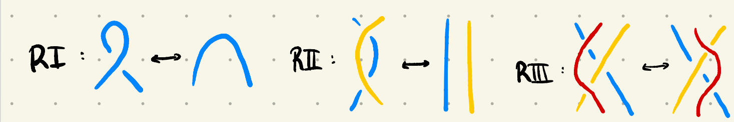
2. Find:

a) A Knot S such that $S \neq$ unknot and $S \#$ trefoil is tricolorable.

b) A Knot T such that, if $K =$ , $K \# T$ is tricolorable.

c) A Knot U such that, for all Knots K, $K \# U = K$

3. Consider the "colors" 0, 1, 2, 3, 4, and define their addition as if they were on a clock: whenever their sum goes over 5, we subtract 5.



This is called "addition modulo 5". Here are some examples:

$4+3 \equiv 7 \equiv 7-5 \equiv 2 \pmod 5$.

$2 \cdot 4 \equiv 8 \equiv 8-5 \equiv 3 \pmod 5$.

$-3-4 \equiv -7 \equiv -7+5 \equiv -2 \equiv -2+5 \equiv 3 \pmod 5$.

<u>Definition</u>: Say a link diagram is 5-colorable if there is a way to assign a number from 0 to 4 to each diagram so that at each crossing  we have $2 \cdot a \equiv b + c \pmod 5$.

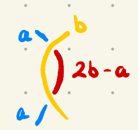
Prove that this is a link invariant by checking the three Reidemeister moves. For example, if  is part of a 5-colorable link, then the same link where we have replaced  by  is also tricolorable, since at both crossings we have: $2 \cdot b \equiv a + 2b - a$

TABLE OF PRIME KNOTS UP TO 8 CROSSINGS