SCGP 3

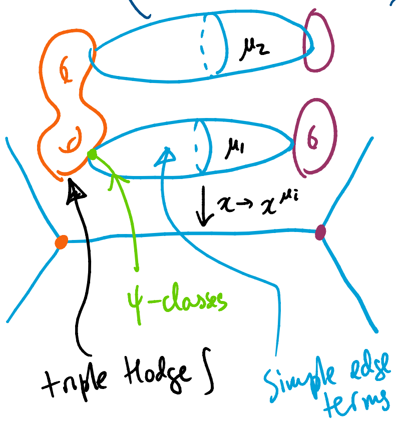
Wednesday, August 24, 2022 12:55 PM

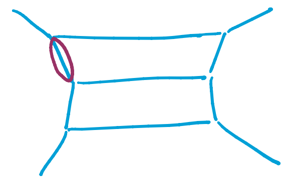
$$\mathrm{GW}/\mathrm{DT}\left(\begin{array}{c} A_r \hookrightarrow X \\ \downarrow \\ B \end{array}\right) = \;?$$

B ← $\mathbb{P}^1$

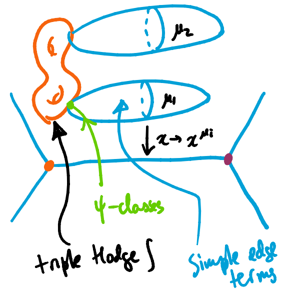
$r = 0$

$X =$

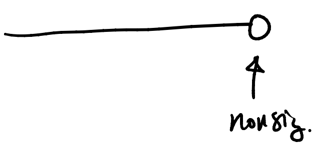




$$\sum_{\mu} \mathrm{edge}(\mu)^{-1} \quad \times$$

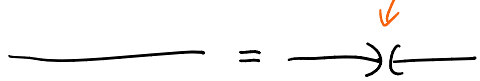


$\times$ same from the other side

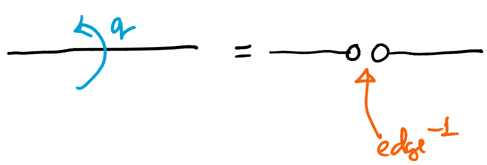


nonsingular b.c.

tangency to $\pi^{-1}(\infty) = D_\infty$

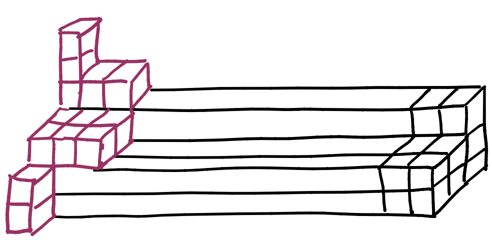


$\mathbb{P}^1$



edge

DT



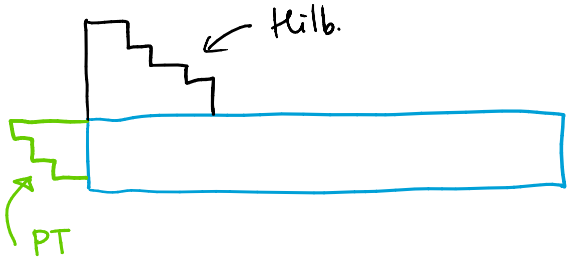
Givental's J- function for Hilb($\mathbb{C}^2$, n)

$\cap$

Nakajima quiver varieties $\in T^*\left(\text{vector space}/G\right)$

Quasimaps

$\parallel$ for A_0 = PT moduli spaces, for A_r, $r>0$ different



solves differential and difference equations

quantum groups

much evolved relative of Hypergeometric function

$1 \in \mathrm{Lie}\, \mathbb{C}_q^* \hookrightarrow B$

$$\sum_{d\geq 0} z^d \prod_{i=1}^{m} \frac{(b_i)_d}{(a_i)_d}$$

$(a)_k = a(a+1)\dots(a+k-1)$

$a_1 = 1$

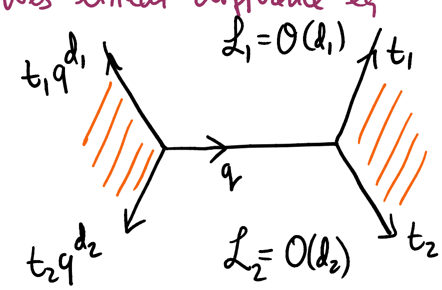
Kähler variable

z^λ, $Q^{\deg}$

equivariant variables t_1, t_2 acts on $\mathcal{L}_1 \otimes \mathcal{L}_2$

solves linear differential eq. in

solves linear difference eq.



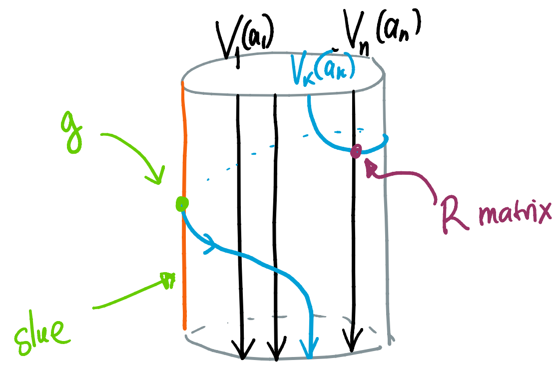
DT, quasimaps makes sense in K-theory

$\forall$ I, quasimaps makes sense in K-theory

q- difference eq. —— " ——

from quantum groups $U_\hbar$

One example of q-diff. conn. from quantum groups $\longrightarrow$ qKZ



$[g\otimes g, R] = 0$

$g =$ Loop rotation $\cdot$ z

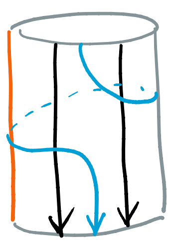
$\quad\quad\quad\quad \| $

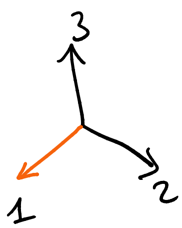
$\quad\quad\quad T^{-1}$

Cartan torus

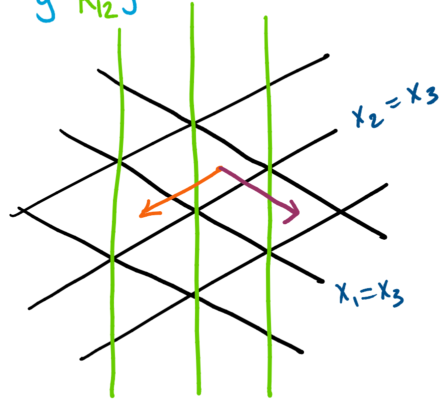
$Ta = qa$

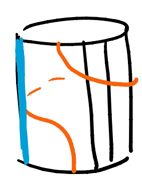
Kähler variables



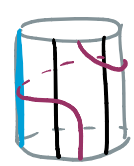


$\bar g^{(1)} R_{12} g^{(1)}$ $\quad R_{12}(a_1/a_2)$





$g^{(1)} \quad R_{13} R_{12}$

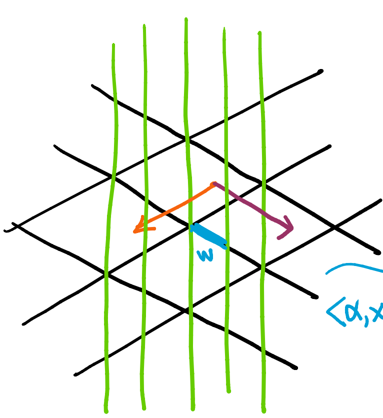


$g^{(2)} \underbrace{g^{(1)} R_{21}\, g^{(1)^{-1}}} R_{23}$

$\bar g^{(2)} R_{21}\, g^{(2)} R_{23}$

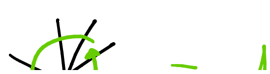
$R_{21}\, g^{(2)} R_{23}$

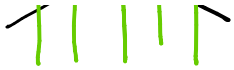
$g^{(i)}$ = acts in the ith tensor factor



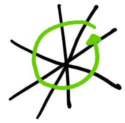
$\longrightarrow B_w(q^n x^\alpha)$

$\langle\alpha, x\rangle + n = 0$





K_{21} g $\cdots$ K_{23}

 $= 1$

λ, $\langle \alpha, \lambda \rangle \in \mathbb{Z}$ $\forall \alpha$ translation by λ = conjugation by $\mathcal{O}(\lambda)$ of all B_w

$\rightsquigarrow$ commuting q-difference operators

transition functions for a vector bundle $\mathbb{C}^{\text{rank}} / q^{\text{shifts}}$.

for $U_\hbar(\hat{\mathfrak{g}})$ $\longrightarrow$ 1) qKZ equation

depends on $z \in \text{Cartan}(\mathfrak{g})$

2) universally, $\exists$ q-difference connection in z that commutes with qKZ

operators $\in U_\hbar(\hat{\mathfrak{g}})$

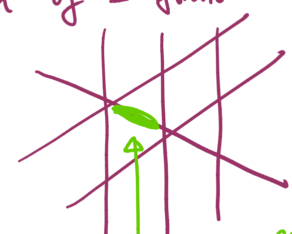
act on any module

$\hookrightarrow V_1(a_1) \otimes \ldots \otimes V_n(a_n)$

qKZ.

Etingof - Varchenko

when $\mathfrak{g}$ = finite-dimensional



$\langle \alpha, x \rangle + n = 0$

$\alpha \in$ root of $\mathfrak{g}$

every wall corresponds to a root $U_\hbar(\mathfrak{sl}_2) \hookrightarrow U_\hbar(\hat{\mathfrak{g}})$

$\cup\!\!\!\shortmid$

B_w quantum dynamical Weyl group

more generally $\longrightarrow$ [O. - A. Smirnov]

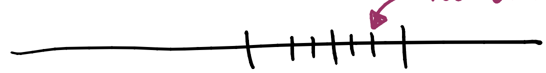
$U_\hbar(\hat{\hat{\mathfrak{gl}}}(1))$, need roots of $\hat{\mathfrak{gl}}(1)$

b

$\pm 1, \pm 2, \pm 3, \ldots$

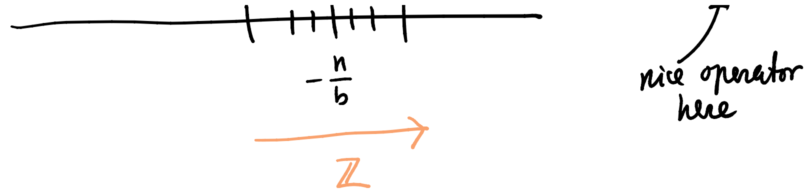
$\bigcup_{b,n} \{ bx + n = 0 \}$

root subalgebra $U_\hbar(\hat{\mathfrak{gl}}(1)) \hookrightarrow U_\hbar(\hat{\hat{\mathfrak{gl}}}(1))$



nice operator

$$\bigcup_{b,n} \{ bx+n=0 \}$$



There are the q-difference equations in the Kähler variables

Logic of the proof:

1) find qKZ among q-difference equations in equivariant variables

2) use commutation with + a few of the other things we know to conclude

$\bigsqcup_{n\geq 0}$.

$K(\mathrm{Hilb}(\mathbb{C}^2, \mathrm{pts})) \curvearrowleft$ module over quantum group

$\otimes$

$K(\text{— '' —})$

$\underbrace{\qquad\qquad\qquad}$

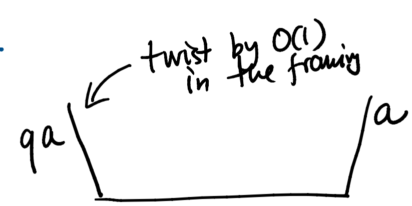
$K(\text{ideal} \oplus \text{ideal}) \quad$ fixed locus $\begin{pmatrix} a & 0 \\ 0 & 1 \end{pmatrix} \leftarrow$ shifts in a qKZ

$\int$

rank 2 framed sheaves on $\mathbb{C}^2$ or $\mathbb{P}^2$ $= M(2)$ $\quad$ $w=2$

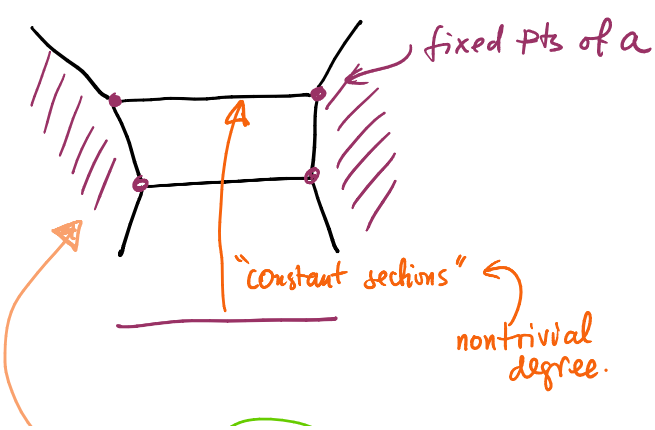
$$\mathcal{F}\big|_{\mathbb{P}^2\setminus\mathbb{C}^2} \simeq \mathcal{O}^{\oplus 2}_{\mathbb{P}^2\setminus\mathbb{C}^2} \qquad GL(2)$$

for Nakajima varieties: $\mathrm{Hilb}(\mathbb{C}^2, n) = M(\underset{v}{n}, \underset{w}{1})$



qKZ $\qquad (Z \otimes 1)\, R_{12}(a)$

Kähler variables enter as monomials



degrees of constant curves

degrees of constant curves

nontrivial degree.

ambient

$$\mathrm{Stab}: \quad K(\mathrm{Hilb}) \otimes K(\mathrm{Hilb}) \longrightarrow K(M(2))$$

so that if I stick these classes in, all contribution cancel except constant curves.

$$R \approx \mathrm{Stab}^T \circ \mathrm{Stab}$$

$$\mathrm{Stab}^T = \mathrm{Unstab}^{-1}$$

need some argument for this vanishing

in cohomology easy [99c]

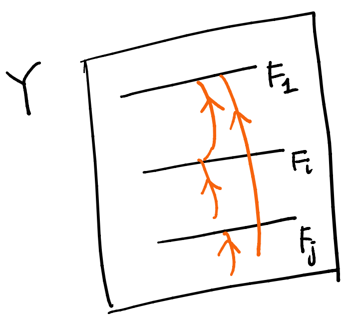
vanishing $\Longleftarrow$ rigidity

properness, $\Rightarrow$ the result is a polynomial in a

degree bound $\Rightarrow$ the result has negative degree in a

repelling

Stab is supported on the full attracting manifold $\subset Y \times Y^a$



$a \to 0$

for degree bounds $\deg\left(\mathrm{Stab}\Big|_{F_j \times F_i}\right) \subset$ fixed polygon.

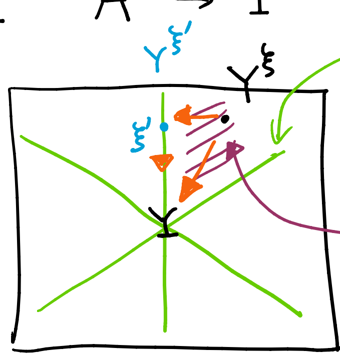
Newton polygon

<u>More abstractly</u>

torus

$$A \hookrightarrow Y$$

Lie A



weights in N_{Y/Y^A}

or $\xi \in \mathrm{Lie}\, A$ such that $Y^\xi \neq Y^A$.

a choice of attracting/repelling directions

$$Y^A \longrightarrow Y$$

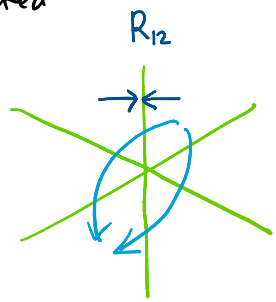
iso after localization.

with some assumptions on Y

$Y^{\cdots} \longrightarrow Y$ localization.

with some assumptions on Y can be constructed

supported on the full attracting set and satisfying some degree condition



$M(r)$

rank 2 sheaves

$r = r_1 + r_2 + r_3$

$A = (\mathbb{C}^*)^3$ acts.

$$M(r)^A = M(r_1) \times M(r_2) \times M(r_3)$$

more generally for Nakajima quiver varieties $w = \sum w^{(i)}$