Homework #2

MATH 2030 Summer A 2021

Due: May 10, 2021

Problem 1

For each of the following differential equations, find all equilibrium points and classify them as stable, unstable, or semistable. The find the requested asymptotic limit for a solution to the equation satisfying an initial condition. You do not have to solve any of the equations to solve this problem.

(i) $\frac{dy}{dt} = y(y-1)(y-3)$; Find $\lim_{t\to\infty} y(t)$ if $y(0) = 2$.

(ii) $\frac{dy}{dt} = e^y(y-2)^2$; Find $\lim_{t\to\infty} y(t)$ if $y(0) = 3$.

(iii) $\frac{dy}{dt} = \cos(y)$; Find $\lim_{t\to\infty} y(t)$ if $y(0) = \pi/2$.

Problem 2

Determine whether each of the following vector fields $\mathbf{v} : \mathbb{R}^2 \to \mathbb{R}^2$ given in components by

$$\mathbf{v}(x, y) = (v_1(x, y), v_2(x, y))$$

is conservative, and if so find its scalar potential.

(i) $\mathbf{v}(x, y) = (3x^2 + y^2, 2xy + 3y^2)$

(ii) $\mathbf{v}(x, y) = (\log(x^2 + y^2), \log(x^2 + y^2)), \quad x > 0, y > 0.$

Problem 3

Find a value b such that the following equation is exact, and find its solution

$$(ye^{2xy} + x) + bxe^{2xy}y' = 0.$$

Problem 4

(i) Consider the equation

$$x^2y^3 + x(1 + y^2)y' = 0$$

. Show that it is not exact, but becomes exact after multiplying by the integrating factor $\mu(x, y) = 1/(xy^3)$. Then solve the equation.

Problem 5

Solve the following initial value problems

(i) $\frac{dy}{dt} + 2ty = sin(t)e^{-t^2}, \; y(0) = 1$

(i) $\frac{dy}{dt} + \frac{y}{t} = \frac{1}{\sqrt{1-t^2}}$ for $0 < t < 1, \; y(1/\sqrt{2}) = 0$.