Homework #8

MATH 2030 Summer A 2021

Due: June 14, 2021

Problem 1

Calculate the inverse laplace transform of the following functions

i) $Y(s) = \frac{e^{-2s}(2s+2)}{s^2+2s+5}$.

ii) $Y(s) = \frac{8s^2-4s+12}{s^3+4s}$.

iii) $Y(s) = \frac{1}{(s+1)^2(s^2+4)}$. For this problem, write your answer using the convolution theorem.

Problem 2

For the following initial value problems, find the impulse response of the system and use this to write the general solution to the IVP as an integral.

i) $y'' + 9y = \cos(t); \qquad y(0) = y'(0) = 0.$

ii) $y'' + my' + ky = g(t); \qquad y(0) = 0, y'(0) = 0$ assuming $k > 0, m \geq 0, m < 2\sqrt{k}$. (Recall the related question on the midterm exam).

iii) $y^{(4)} + 5y'' + 4y = g(t); \qquad y(0) = y'(0) = y''(0) = y'''(0) = 0.$

Problem 3

Prove the following facts about convolution.

i) $f * g = g * f$.

ii) $f * (g * h) = (f * g) * h$.

iii) $\delta * f = f$.

Problem 4

The result in Homework #7, Problem 6 about the Laplace transform of $J_\nu(x)$ is true whenever $\nu > -1$. Use this to show that for any $\nu \in [0, 1)$ we have

$$(J_\nu * J_{-\nu})(x) = \sin(x).$$

Problem 5

Use the Laplace transform to solve the following inhomogeneous system of ODEs with discontinuous forcing function.

$$\begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix}' = \begin{pmatrix} 1 & -3 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} y_1(t) \\ y_2(t) \end{pmatrix} + u_2(t) \begin{pmatrix} 1 \\ -2 \end{pmatrix}; \qquad \vec{y}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$