Mathematics V1205y
Calculus IIIS/IVA

Midterm Examination #1

February 28, 2000
11:00 am – 12:15 pm

1. Evaluate the iterated integral $\int_0^9 \int_{\sqrt{y}}^3 \sin(\pi x^3)\, dx\, dy$.

2. Find the volume below the cone $z = r$ and above the unit disk centered at $r = 1$, $\theta = 0$.

3. Let E be the spherical box defined by

$$E = \{(\rho, \theta, \phi) \,:\, 0 \le \rho \le 1,\, 0 \le \theta \le \pi/4,\, 0 \le \phi \le \pi/2\}.$$

Sketch E carefully on axes labelled x, y, z. Then evaluate $\iiint_E \rho \sin\theta\, dV$.

4. Find the center of mass of the lamina that occupies the part of the unit disk $x^2+y^2 \le 1$ in the first quadrant, if the density at any point is proportional to its distance from the x-axis.

5. Find the surface area of that part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the x, y-plane.

6. Let $f(x)$ and $g(y)$ be continuous functions which are *odd* and *even* respectively: that is, $f(-x) = -f(x)$ while $g(-y) = g(y)$. For each of the following integrals, say whether it is ≤ 0, ≥ 0, $= 0$, or impossible to tell. Give a very brief reason.

(a) $\int_0^1 \int_0^{x+1} \left(2f(x)g(y) - f(x)^2 - g(y)^2\right) dy\, dx$;

(b) $\int_0^1 \int_{y-1}^{1-y} f(x)\, g(y)\, dx\, dy$;

(c) $\int_0^1 \int_{-1}^1 f(x)\, g(y)\, dy\, dx$.

Some possibly useful integrals:

$$\begin{aligned}
\int \sin^2 u\, du &= \tfrac{1}{2}u - \tfrac{1}{4}\sin 2u + C \\
\int \cos^2 u\, du &= \tfrac{1}{2}u + \tfrac{1}{4}\sin 2u + C \\
\int \tan^2 u\, du &= \tan u - u + C \\
\int \sin^3 u\, du &= -\tfrac{1}{3}(2 + \sin^2 u)\, \cos u + C \\
\int \cos^3 u\, du &= \tfrac{1}{3}(2 + \cos^2 u)\, \sin u + C \\
\int \tan^3 u\, du &= \tfrac{1}{2}\tan^2 u + \ln|\cos u| + C
\end{aligned}$$