Mathematics V1205y
Calculus IIIS/IVA

Answers to Midterm Exam #1

February 28, 2000
11:00 am – 12:15 pm

1. $\int_0^9 \int_{\sqrt{y}}^3 \sin(\pi x^3)\,dx\,dy = \int_0^3 \int_0^{x^2} \sin(\pi x^3)\,dy\,dx = \int_0^3 x^2 \sin(\pi x^3)\,dx = -\frac{1}{3\pi}\cos(\pi x^3)_0^3 = 2/(3\pi)$.

2. The disk is bounded by $r = 2\cos\theta$, so the volume is $\int_{-\pi/2}^{\pi/2} \int_0^{2\cos\theta} r\,r\,dr\,d\theta$
$= \int_{-\pi/2}^{\pi/2} (r^3/3)_{r=0}^{r=2\cos\theta} d\theta = \int_{-\pi/2}^{\pi/2} \frac{8}{3}\cos^3\theta\,d\theta = \frac{8}{9}\Big((2+\cos^2\theta)\sin\theta\Big)_{-\pi/2}^{\pi/2} = 32/9.$

3. See diagram below. And $\iiint_E \rho\sin\theta\,dV = \int_0^{\pi/4}\int_0^{\pi/2}\int_0^1 \rho\sin\theta\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$
$= \Big(\int_0^{\pi/4}\sin\theta\,d\theta\Big)\Big(\int_0^{\pi/2}\sin\phi\,d\phi\Big)\Big(\int_0^1\rho^3\,d\rho\Big) = (1-\sqrt{2}/2)\cdot 1\cdot 1/4 = 1/4 - \sqrt{2}/8.$

4. Density is cy for some constant c, so mass is $m = \iint_D cy\,dy\,dx = \int_0^{\pi/2}\int_0^1 cr\sin\theta r\,dr\,d\theta$
$= c/3\int_0^{\pi/2}\sin\theta\,d\theta = c/3.$
Also y-moment is $M_y = \iint_D x\,cy\,dy\,dx = \int_0^{\pi/2}\int_0^1 r\cos\theta\,cr\sin\theta\,r\,dr\,d\theta$
$= c/4\int_0^{\pi/2}\sin\theta\cos\theta = c/16\int_0^{\pi/2} 2\sin 2\theta\,d\theta = c/16(-\cos 2\theta)_0^{\pi/2} = c/8;$
and x-moment is $M_x = \iint_D y\,cy\,dy\,dx = \int_0^{\pi/2}\int_0^1 cr^2\sin^2\theta\,r\,dr\,d\theta = c/4\int_0^{\pi/2}\sin^2\theta\,d\theta$
$= c/4(\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta)_0^{\pi/2} = \pi c/16$. So center of mass is $(M_y/m, M_x/m) = (3/8, 3\pi/16)$.

5. Boundary of plane region D where paraboloid lies above plane is where $0 = 4 - x^2 - y^2$, circle of radius 2. So surface area is $\iint_D \sqrt{(\partial z/\partial x)^2 + (\partial z/\partial y)^2 + 1}\,dA = \iint_D \sqrt{4x^2+4y^2+1}\,dA = \int_0^{2\pi}\int_0^2 (4r^2+1)^{1/2} r\,dr\,d\theta = 2\pi\cdot\frac{1}{12}\int_0^2 \frac{3}{2}(4r^2+1)^{1/2} 8r\,dr = \pi/6\Big((4r^2+1)^{3/2}\Big)_0^2 = \pi/6(17\sqrt{17}-1)$.

6. (a) ≤ 0 by comparison to 0, since integrand $= -(f-g)^2 \leq 0$; (b) $= 0$ by symmetry: since $f(x)\,g(y)$ is an odd function of x, the triangle to the left of the y-axis makes a contribution equal and opposite to that of the triangle to the right; (c) impossible to tell: take e.g. $f(x) = x$ and $g(y) = y^2$ or $-y^2$ to get two different signs.